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By RICHARD CARR.

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PREFACE.

A Great many have wrote upon this Subject, and most have met with deserved Success; but how far this may merit the same, Time alone must determine. I only wish it may meet with impartial Judges, who will undoubtedly give it a Character equal to its Deserts, and then I have Reason to believe it will answer the End designed; that is, to be a Guide to them that want to attain a Knowledge of those useful and entertaining Studies, and an agreeable Companion for such as have made some Progress in them. The fundamental Rules of Arithmetic are laid down so plain, that the meanest Capacity, with a little Application, may understand them; and when I come to the Rule of Three, I make use of a few Characters which the Reader may soon be Master of; and then may proceed with Pleasure. What Rules are omitted, have their Dependance on the Rules of Proportion, which are fully explained; and as most Questions in Practice are done with much Ease and Certainty, by the Multiplication here made use of; I hope the Want of them will be no Defect. I have very plainly and intelligibly delivered the Algebraical Part, and exemplified the Rules with a Collection of Questions, answered in a plain and familiar Manner. I have not notified in the Margin when you are to make Use of a Step, or an absolute Number; but the Step following will determine which it is.

I have taken Care to keep this Treatise as free as possible from the Errors of the Press; but if there should be any, I hope the Reader will correct and excuse them. The Questions are mostly answered numerically, being the most easy Way to initiate the Use of Algebra to Learners, which when understood, they may use the literal Way with Pleasure.

NUMERATION

TEaches to place, or give the juſt Value, to any Number demanded; to do which, obſerve the following Table.

The TABLE.

								1	Units.
							2	1	Tens.
						3	2	1	Hundreds.
					4	3	2	1	Thouſands.
				5	4	3	2	1	T. of Th.
			6	5	4	3	2	1	C. of Th.
		7	6	5	4	3	2	1	Millions.
	8	7	6	5	4	3	2	1	T. of M.
9	8	7	6	5	4	3	2	1	C. of M.
9	8	7	6	5	4	3	2	1	

In the Table above you ſee how each Place exceeds the former ten times, increaſing towards the left Hand. The firſt is the Place of Units, the ſecond of Tens, the third of Hundreds, &c.

ADDITION

Is the gathering several Numbers into one, which is then called the Sum or the Aggregate; as 6 and 8 are 14.

* Addition begins at the right Hand, and adds the particular Sums of the several Rows underneath every one in its own Place.

EXAMPLES.

Ex. 1st.	Ex. 1d.	Ex. 3d.
2743	2435	2374
5427	1437	5615
6514	6214	8731
3215	3715	6425
6732	7214	7431
5475	3746	5762
30106	24761	36338

In the first *Example*, the Sum of the first Row is 26; I place down the 6 under itself, and carry 2, being the Number of the Tens to the next Row, and find the Sum of it to be twenty, then I place down a Cypher, and carry two to the next Row, and find the Sum to be 31. I place down the 1, and carry 3 to the last Row, and the Sum is 30, which I place down, and it is done. And so of the rest.

ADDITION of MONEY.

N. B. Four Farthings is one Penny, Twelve Pence one Shilling, and Twenty Shillings one Pound.

EXAMPLES.

l.	s.	d.
27	14	11 ½
15	16	4 ¼
72	17	3 ¾
56	18	10 ¼
32	11	9 ¼
64	16	7 ¾
37	14	6 ½
67	19	2 ¼
376	09	7 ¾

l.	s.	d.
374	17	6 ½
542	15	11 ¼
364	16	7 ¾
273	18	3 ¼
614	15	7 ¼
364	19	3 ½
764	13	5 ¼
527	11	10 ½

In the first *Example.* The Farthings are 15, which is three Pence three Farthings, put down ¾ under the Farthings, and carry three to the Pence place, and the Sum of the Pence is 55, which is 4 Shillings and 7 Pence; place the 7 under the Pence, and carry 4 to the Shillings, and the Sum is 129 Shillings, which is 6 Pounds 9 Shillings; place down the 9, and carry 6 to the Pounds, then proceed as in the last Examples of whole Numbers.

l.	s.	d.	l.	s.	d.
375	17	4 $\frac{1}{2}$	742	14	3 $\frac{1}{2}$
654	14	9 $\frac{1}{4}$	567	15	7 $\frac{3}{4}$
276	11	4 $\frac{1}{2}$	254	18	6 $\frac{1}{4}$
346	14	7 $\frac{3}{4}$	675	17	8 $\frac{1}{2}$
214	16	2 $\frac{1}{2}$	792	15	7 $\frac{3}{4}$

Addition is proved by making a ſecond Addition of the ſame Sums, omitting the top Line; then adding the Sum to the top Line; if it makes the Sum of the whole it is right.

N. B. Addition of Time, Weight, Meaſure, &c. &c. is performed in the ſame Manner, only obſerving how many of the inferior makes one of the next, aſcending by the Tables after Diviſion.

SUBTRACTION

IS the taking a ſmall Number from a large one, and the Number found is called the Remains, the Exceſs or the Difference. As from 12 take 4, and there remains 8.

Examples.

EXAMPLES.

Example 1st.		Example 2d.	
From	37 427	From	37 427
Take	21 314	Take	14 678
Remains	16 113	Remains	22 749
Proof	37 427	Proof	37 427

N. B. In the firſt *Example*, I take 4 from 7, and there remains 3, and ſo place it down, and proceed to the End of that Example.

But in the ſecond *Example*, I cannot take 8 from 7, ſo I borrow 10 and put it to the 7, and it makes 17, then 8 from 17, and there reſts 9, which I place down, and carry one that I borrowed to the 7, and it makes 8; then 8 from 2 I cannot, but I borrow 10, and add it to the 2, and it makes 12; then 8 from 12, and there remains 4, which place down; and ſo proceed to the End, remembring to carry one to the next Figure when you borrow the 10.

If you add the Remains to your leaſt given Number, the Sum will be the greater Number given, which is the Proof.

	l. s. d.		l. s. d.
From	37 : 17 : 11 $\frac{1}{2}$	From	274 : 19 : 03 $\frac{3}{4}$
Take	24 : 13 : 10 $\frac{1}{4}$	Take	175 : 16 : 04 $\frac{1}{2}$
Differ.	13 : 4 : 1 $\frac{1}{4}$	Remains	99 : 02 : 11 $\frac{1}{4}$
Proof	37 : 17 : 11 $\frac{1}{2}$	Proof	274 : 19 : 03 $\frac{3}{4}$

N. B. What you carry at in *Addition*, you borrow at in *Subtraction.*

A Gentleman bought an Estate, valued at 7427 *l.* 17 *s.* 11 *d.* and his Cash is only 1498 *l.* 18 *s.* 11 *d.* $\frac{3}{4}$, how much must he send for to his Banker to be enabled to pay for it? Place the Sums thus:

	l. s. d.
Value of the Estate - -	7427 : 17 : 11
The Cash at home - -	1498 : 18 : 11 $\frac{3}{4}$
This comes from the Banker	5928 : 18 : 11 $\frac{1}{4}$
Proof	7427 : 17 : 11

N. B. The Cash brought from the Banker, being added to the Cash at home, must be the Value of the Estate.

What Sum of Money added to 374 *l.* 11 *s.* 5 *d.* $\frac{1}{2}$, will make it 1000 *l.* Place it as follows:

l. s. d.	
1000 : 00 : 00	
374 : 11 : 05 $\frac{1}{2}$	
625 : 08 : 06 $\frac{1}{2}$	this is the Sum you must add.
1000 : 00 : 00	Proof.

What

What is the Difference betwixt 207*l.* 14*s.* 2*d.* and 176*l.* 19*s.* 11*d.*

	l.	*s.*	*d.*
	207 :	14 :	02
Least Sum	176 :	19 :	11 ½
Difference	30 :	14 :	02 ½
Proof	207 :	14 :	02

A Steward received the following Sums for his Lord; from *A.* 172 *l.* 17 *s.* 11 *d.* ½; from *B.* 160 *l.* 14 *s.* 8 *d.* from *C.* 270 *l.* 11 *s.* 4 *d.* from *D.* 120 *l.* 11 *s.* 6 *d.* from *E.* 192 *l.* 11 *s.* 8 *d.* ½. And he sent to his Lord Bank Notes, Value 407 *l.* 17 *s.* 11 *d.* Paid Workmen 62 *l.* 17 *s.* 11 *d.* Paid the Wine-Merchant 42 *l.* 17 *s.* 00 *d.* His own Salary for a Quarter 12 *l.* 10 *s.* 00 *d.* What remains of his Lord's in his Hands?

From *A.*	172 : 17 : 11½	Sent in Notes	407 : 17 : 11
From *B.*	160 : 14 : 8	P. Workmen	62 : 17 : 11
From *C.*	270 : 11 : 4	W. Merchant	42 : 17 : 00
From *D.*	120 : 11 : 6	Salary	12 : 10 : 00
From *E.*	192 : 11 : 8½		
		T. laid out.	526 : 02 : 10
Tot. rec.	917 : 07 : 02	Money in Hand.	391 : 04 : 04
Disburst.	526 : 02 : 10		
Ballance,	391 : 04 : 04	Proof	917 : 07 : 02

If the Tower of *London* was built by the *Conqueror* in the Year 1077, how many Years is it since, this Year being 1750.

The

The preſent Year - -	1750
The Year when the Tower was built	1077
Years ſince the Tower was built	673
Proof	1750

If the Time any Building has ſtood be added to the Year it was built in, it makes the preſent Year.

An Apprentice was bound for 7 Years, but he runs away at the end of 6 Years, 6 Months, 6 Weeks, 6 Days, 6 Hours, 6 Minutes, the Queſtion is, how long he had to ſerve?

	Y.	M.	W.	D.	H.	M.
The Time he was to ſerve is,	7	0	0	0	0	0
The Time he did ſerve was,	6	7	2	6	6	6
The Time he had to ſerve	0	5	1	0	17	54
Proof	7	0	0	0	0	0

N. B. The 6 Weeks is 1 Month and 2 Weeks, and 13 Months is one Year.

MULTIPLICATIO

IS that by which we find the Increaſe or Amount of any Number, being ſo many Times taken as there are Units in another Number, and this Increaſe or Amount is called the Product. The Factus, the Rectangle, or the Plain; and the Numbers produc-ing

The TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Find any Number in the Top of the Table, and the Number you would multiply it by, in the Side, and in the Square where they meet is the Product. Suppose you would multiply 7 by 8, find the 8 at the Top, and the 7 in the Side, and in the Square where they meet stands 56 the Product, and so of any other.

Mind to have the Table off by Heart, before you go any farther.

Multiply	674274	This is the Multiplicand.
By	6	This the Multiplier.
	4045644	This is the Product, &c.

In the above Example I ſay, 6 times 4 is 24; put down 4 and carry 2 to the next Place, ſaying 6 times 7 is 42, and 2 I carry is 44; put down 4 and carry 4. And proceed in this Manner to the End, putting down the odd Numbers, and carry one for every Ten that you have to the next Place.

Multiply	576274
By	37
	4033918
	1728822
	21322138

N. B. You muſt place the Beginning of each Multiplication under its own Factor, as you may obſerve in the above Example; when I began to multiply by 3, I put the firſt Figure of its Product, being 2, under itſelf, and then proceeded as before, and the two Multiplications added into one Sum, is the Product.

Multiply	674076
By	346
	4044456
	2696304
	2022228
	233230296

Multiply

Multiply	47247
By	3004
	188988
	14174100
	141929988

In the above Example, the Cyphers only fill up the Places under which they ſtand, and then you begin your Multiplication by the 3.

Multiply	674264
By	300
	202279200

Multiply	721462
By	7400
	288584800
	5050234
	533881800

N. B. If your Multiplier have Cyphers to the right Hand, it is only multiplying by the Figures, and adding the Cyphers to the Product.

Multiply	67424
By	1000
	67424000

To multiply by an Unit with any Number of Cyphers; it is only adding the Cyphers to the Multiplicand.

Multiply	74624	Multiply	5742
By	15		17
	1119360		97614

This Multiplication is performed by taking in the back Figure, thus, as in the first Example, I ſay 5 times 4 is 20, put down 0 and carry 2, then 5 times 2 is 10, and 2 I carry is 12, and the back Figure 4, is 16, put down 6 and carry 1; then 5 times 6 is 30, and 1 I carry is 31, and 2 the back Figure is 33, put down 3 and carry 3; then 5 times 4 is 20, and 3 is 23, and the back Figure 6 is 29, put down 9 carry 2; then 5 times 7 is 35 and 2 is 37, and the back Figure 4 is 41, put down 1 and carry 4. Now as the Multiplication by 5 is ended, I add the 4 I carry to the 7 and it makes 11, which I put down, and the Work is ended; and ſo of any other.

Multiply	54762	Multiply	75642
By	19	By	18
	1040478		1361556

Multiplication by two back Figures is performed by the former Rule.

EXAMPLES.

Multiply 742657	Multiply 67542
By 1714	1619
10397198	1283298
12625169	1080672
1272914098	109350498

Multiplication may be performed by Factors, thus. Take any two or three Numbers, &c. which when multiplied into one another, will produce the Multiplier given; and multiply the Multiplicand by one of them, and that Product by the other, &c. and the last Multiplication will be the desired Product.

EXAMPLES.

Multiply 74276 by 24: The Factors are 4 and 6.

```
74276
    6
------
445656
     4
-------
1782624
```

Multiply 67424 by 48. The Factors are 8 and 6.

```
67424
    8
------
539392
     6
-------
3236352
```

N. B.

N. B. The last Product had been the same, if I had taken any other Numbers; which when multiplied together, would have made the Multiplier; as 3, and 8, in the first Example. And 4, 6, and 2 in the second.

Multiplication of Compound Quantities.

If one Pound of Tea cost 5 *s.* 9 *d.* what will eight Pounds cost?

$$\begin{array}{r} 5 : 9 \\ 8 \\ \hline l.\ 2 : 6 : 0 \end{array}$$

I say 8 times 9 is 72, which is 72 Pence, that is 6 Shillings, put a Cypher under the Pence place, and carry 6, saying 8 times 5 is 40, and 6 I carry is 46, which is 46 Shillings, equal to 2 *l.* 6 *s.* 0 *d.* the Answer.

If one Yard of Velvet cost 1 *l.* 4 *s.* $6\frac{1}{2}$ *d.* What will 9 Yards cost at that rate?

$$\begin{array}{r} 1 : 4 : 6\frac{1}{2} \\ 9 \\ \hline 11 : 0 : 10\frac{1}{2} \end{array}$$ The Answer.

For 9 Halfpence is 4 *d.* $\frac{1}{2}$; I set down the $\frac{1}{2}$, and carry 4 to the Pence place, saying 9 times 6 is 54, and 4 I carry is 58 Pence, which is 4 *s.* 10 *d.* put down the 10 and carry 4, saying 9 times 4 is 36, and 4 I carry is 40 Shillings, put down a Cypher in the Shillings place, and carry 2 to the Pounds place, saying 9 times 1 is 9, and 2 I carry is 11 Pounds,

 which

which I put down, and the Anſwer is 11*l.* 0*s.* 10*d.* $\frac{1}{2}$. And ſo of all the reſt of the Examples.

If 1 Yard coſt 3*s.* 7*d.* $\frac{1}{4}$, what will 24 Yards coſt at that rate?

$$\begin{array}{r} 3 : 7\frac{1}{4} \\ 3 \\ \hline 10 : 9\frac{3}{4} \\ 8 \\ \hline 4 : 6 : 6 \end{array}$$

N. B. The Factors are 3 and 8, or 4 and 6; ſo that multiplying firſt by 3, and that Product by 8, gives the Anſwer.

If 1 Pound of Tea coſt 7*s.* 4*d.* $\frac{1}{2}$, what will 56 Pounds coſt at that rate?

$$\begin{array}{r} 7 : 4\frac{1}{2} \\ 8 \\ \hline 2 : 19 : 0 \\ 7 \\ \hline 20 : 13 : 0 \end{array}$$

In anſwering Queſtions of this Kind, take any two Numbers, which when multiplied together will produce the Number given, and multiply the Price by one of the Factors, and that Product by the other, and the laſt Product will be the Anſwer, as in the two laſt Examples. In the firſt I multiply the Price of 1 Yard by 3, which gives me the Price of 3 Yards, then I multiply the Price of 3 Yards by 8, and it gives me the Price of 8 times 3 Yards, which is 24 Yards. If I had multiplied the Price of 1 Yard by 4, and that Product by 6, it would have been the ſame

fame. Anſwer. And in the ſecond Example, I multiply firſt by 7, and then by 8 for the ſame Reaſon.

If 1 Yard coſt 2 *s.* 3 *d.* what will 60 Yards coſt at that Price?

2 : 3	
10	
1 : 2 : 6	This is the Price of 10 Yards.
6	
6 : 15 : 0	The Price of 60 Yards.

If a parcel of Cloth coſt 8 *l.* 16 *s.* 5 *d.* what will 336 Parcels coſt at that rate?

8 : 16 : 5	
10	
88 : 4 : 2	The Price of 10 Parcels.
10	
882 : 01 : 8	The Price of 100
3	
2646 : 05 : 0	The Price of 300
264 : 12 : 6	The Price of 30
52 : 18 : 6	The Price of 6
2963 : 16 : 0	The Price of 336

Firſt, I multiply the Price of 1 Parcel by 10, which gives me the Price of 10 Parcels, then I multiply that Product by 10, and it gives the Price of 10 times 10 Parcels, or 100; which Product multiplied

plied by 3 gives the Price of 300 Parcels. Then multiplying the Price of 10 Parcels by 3, it gives the Price of 30 Parcels, which place under the Price of 300 Parcels. Lastly, multiply the Price of one Parcel by 6, and place it under the last, then those 3 Sums added together gives the Answer equal, 2963 *l.* 16 *s.*

If 1 Gallon of Rum cost 8 *s.* 6 *d.* what will 6742 cost?

s. d.	
8 : 6	The Price of 1 Gallon.
10	
l. 4 : 5 : 0	The Price of 10 Gallons.
10	
42 : 10 : 0	The Price of 100 Gallons.
10	
425 : 00 : 0	The Price of 1000 Gallons.
6	
2550 : 00 : 0	The Price of 6000 Gallons.
297 : 10 : 0	The Price of 700 Gallons.
17 : 00 : 0	The Price of 40 Gallons.
00 : 17 : 0	The Price of 2 Gallons.
2865 : 07 : 0	The Price of 6742 Gallons.

This Method of finding the Value of any Number of Yards, Gallons, Ells, Pounds, &c. at any Price *per* Yard, Gallon, &c. is of great Use for finding the Value in a short and easy manner, and very often exceeds the Method of Practice; and will be of ex-

cellent Use for such as buy or sell by Retail, as no Multiplication can be easier than multiplying by 10.

If a Hogshead of Tobacco cost 3 l. 8 s. 4 d. $\frac{1}{2}$, what is the Value of 432 Hogsheads?

l.	s.	d.	
3 :	8 :	4$\frac{1}{2}$	
		10	
34 :	03 :	09	The Price of 10.
		10	
341 :	17 :	06	The Price of 100.
		4	
1367 :	10 :	00	The Price of 400
102 :	11 :	03	The Price of 30
6 :	16 :	09	The Price of 2
1476 :	18 :	00	The Price of 432

Cross Multiplication; or, the Method of squaring Dimensions.

This differs very little from Compound Multiplication, only in placing the Figures of your Multiplicand and Multiplier. Examples will make it very easy. Remember to carry 1 at every 12 to the next superior Name, excepting when you multiply Feet by Feet, and then you carry 1 at every 10, as in common Multiplication.

EXAMPLES.

EXAMPLES.

If a Board be 1 Foot 4 Inches broad, and 13 Feet long, how many Feet is contained therein? Place it thus.

```
F.   I.
13 : 0      Length.
     1 : 4  Breadth.
-----------
 4 : 4 : 0
13 : 0
-----------
17 : 4 : 0  The Anſwer.
```

If a Board be 8 Feet 6 Inches long, and 3 Feet 4 Inches broad, what is the ſuperficial Content?

```
F.  I.  P.
8 : 6
    3 : 4
-----------
 2 : 10 : 0
25 :  6 : 0
-----------
28 :  4 : 0  The Anſwer.
```

```
               F. I. P. S. T.       F. I. P. S. T.
Multiply       8 : 4 : 7 : 3 : 5  by 4 : 2 : 6 : 4 : 3
   Place it thus                    4 : 2 : 6 : 4 : 3
                                  ---------------------
                 2 : 1 : 1 : 9 : 10 : 3
             2 : 9 : 6 : 5 : 1 : 8
         4 : 2 : 3 : 7 : 8 : 6
     1 : 4 : 9 : 2 : 6 : 10
  33 : 6 : 5 : 1 : 8
-----------------------------------------
  35 : 3 : 7 : 7 : 6 : 1 : 5 : 6 : 3
```

Multiply

Multiply 259 Feet 10 Inches and 8 Parts, by 18 Feet 5 Inches and 4 Parts.

	F.	I.	P.	S.	T.
	259 :	10	08		
			18 :	5 :	4
	7 :	2 :	7 :	6 :	8
	108 :	3 :	5 :	4	
First multiplied by Ten	2598 :	10 :	8		
First multiplied by Eight	2079 :	1 :	4		
The Product	4793 :	6 :	0 :	10 :	8

If the Learner mind to carry 1 for every 12 to the next superior Place, setting down the Remains under 12 in its own Place, (excepting when he multiplies Feet by Feet, which he must carry 1 for every 10, as in common Multiplication) he cannot go wrong; for the placing the Multiplicand and Multiplier in this Manner, shews at once the Name of every particular Product, as in the above Example, Parts multiplied by Parts, give Thirds in the Product; that is, 4 times 8 is 32 Thirds, which is 2 Seconds and 8 Thirds; put down the 8 and carry 2, saying, 4 times 10 is 40, and 2 is 42, which is 6 Seconds and 3 Parts, put down the 6 and carry 3; then multiply 259 Feet by 4 Parts, and the 3 Parts that you carry makes 1039 Parts, which divided by 12 gives 86 Inches and 7 Parts remaining; then the Inches divided by 12, gives 7 Feet and 2 Inches remaining, which place down in their own Places, and it is done. So proceed with the rest.

If

If a Board be 2 Feet 6 Inches ſquare, what is the ſuperficial Content?

F.	I.	
2 :	6	
	2 :	6
1 :	3 :	0
5 :	0	
6 :	3 :	0 The Anſwer.

If a Plank be 8 Feet 7 Inches and 6 Parts long, and 4 Feet 6 Inches and 4 Parts broad, how many ſquare Feet is contained in that Plank?

	F.	I.	P.	S.	T.
	8 :	7 :	6		
			4 :	6 :	4
	0 :	2 :	10 :	6 :	0
	4 :	3 :	9 :	0	
	34 :	6 :	0		
The Anſwer	39 :	0 :	7 :	6 :	0

If a Room be 37 Feet 4 Inches and 8 Parts long, and 28 Feet 7 Inches and 6 Parts broad, how many ſquare Feet is contained in that Room?

F.

F. I. P.
28 : 7 : 6 Breadth.
37 : 4 : 8 Length.

		F. I. P.
1 : 7 : 1 : 0 : 0	Top mult. by	00 0 8
9 : 6 : 6 : 0	Top mult. by	00 4 0
200 : 4 : 6	Top mult. by	07 0 0
286 : 3 : 0	Top mult. by	10 0 0
572 : 6 : 0	The last mult: by 2 equals	20 0 0
1070 : 3 : 1 : 0 : 0	Content	37 4 8

N. B. The last multiplied by 2 is the same as if the Top was multiplied by 20, because twice 10 is 20. And the Addition to the right Hand, shews that the several Multiplicators added, is the same as your first Multiplicator or Length. If the Learner postpone this till he is perfect in Division, it will be a great deal more plain and easy.

Let there be a Piece of Timber in Form of a Parallelopipedon, whose length is 8 Foot 6 Inches, and the side of the Square its Base 2 Foot 6 Inches; I demand its Solidity.

2 : 6 Side of the Base.
2 : 6

1 : 3 : 0
5 : 0

6 : 3 : 0
8 : 6

3 : 1 : 6
50 : 0 :

53 : 1 : 6 Solidity.

DI-

DIVISION

IS a Rule by which we discover how many times one Number (called the Divisor) is contained in another (called the Dividend) and the Number found is called the Quotient. And sometimes there is a fourth Number called the Remainder.

N. B. The Quotient so often contains an Unit, as the Dividend contains the Divisor.

EXAMPLES.

Divide 24 Pounds amongst 4 Men.

Place them in this Manner.

4)24(6
24

N. B. 24 is the Dividend, 4 is the Divisor, and 6 is the Quotient, found by dividing the Dividend by the Divisor; saying how many times 4 in 24, which is 6 times, and then multiplying the Quotient Figure by the Divisor, and place the Product under the Dividend which is 24, and nothing remains, and the Quotient is 6 Pounds *per* Man.

If

If 6678 Pounds be equally divided amongſt 9 Men, what is each Man's Share?

```
9)6678(742
  63
  --
   37
   36
   --
    18
    18
    --
     0
```

I ſay how many times 9 in 66, and I find 7 times 9 is 63, which I place under the 66, and ſubtract, and find the Difference 3, to which I bring down the 7, and make it 37; then I find how many times 9 in 37, and it is 4 times 9, which is 36, which I place under 37, and ſubtract, and find the Difference 1, to which I bring down the 8, and make it 18; then how many times 9 in 18, which is 2 times, which I put in the Quotient, and multiply the Diviſor by it, and find it 18, which I place under 18, and find no Remainder; and each Man's Share is 742 Pounds.

Diviſion is the only true way to prove Multiplication; for if one divide the Product by the Multiplier, the Quotient will be the Multiplicand, and dividing the Product by the Multiplicand, the Quotient will be the Multiplier.

So

So is Multiplication the Way to prove Diviſion; for multiplying the Quotient by the Diviſor, the Product produced will be the Dividend. *N. B.* If there be a Remainder it muſt be added to the Product, and the Sum will be the Dividend.

This leads to a ſhort and eaſy Method to be Maſter of Diviſion; as for Example, I multiply 742 by 24. Place it thus:

```
  742           24)17808(742
   24              168
 ----             ----
 2968              100
1484                96
-----             ----
17808               48
                    48
                  ----
                    00
```

I firſt multiply and find the Product 17808, which I divide by 24, and I know the Quotient will be 742; yet I proceed in the Form of Diviſion, multiplying every Quotient Figure into the Diviſor; by this Means you will make the Form of Diviſion familiar.

Multiply 5746	28)160888(5746
By 28	140
45968	208
11492	196
160888	128
	112
	168
	168
	0

Multiply 67432	507)34188024(67432
By 507	3042
472024	3768
3371600	3549
34188024	2190
	2028
	1622
	1521
	1014
	1014
	0

Thus by taking promiscuously any two Numbers, and multiplying them together, and then dividing the Product by one of them, you will have the other in the Quotient; and by doing in this Manner,

ner, you will facilitate Divifion, and fooner become Mafter of it, than by any other Direction.

Multiply	60407
By	2004
	241628
	12081400
	121055628

Divide 121055628 by 60407

```
60407)121055628(2004
      120814
      ------
         241628
         241628
         ------
              0
```

N. B. That every Figure you bring down from the Dividend to the Remainder, if you cannot have the Divifor in it, you muft place a Cypher in the Quotient, as in the above Example: After I can have my Divifor twice in the firft Part of my Dividend, and there remains 241; then I bring down the 6 from the Dividend to it, and it is 2416; but I cannot have my Divifor in it, fo I place a Cypher in the Quotient, and bring down the next Figure 2 to it, and ftill find I cannot have my Divifor in it; fo I place a Cypher in the Quotient, and bring down my laft Figure 8, and find it contains my Divifor 4 times, and there is no Remainder.

A Gentleman left 740 Pounds to be divided amongst 10 Servants. What is each Man's Share?

1|0)74|0(74 Pounds each Man.
74

To divide any Number by 10, is only cutting from the Dividend a Figure to the right Hand, and the Remainder of the Dividend is the Quotient. If your Divisor is 100, cut 2 from the Dividend; if 1000, cut 3 Figures from it, and the Remainder is the Quotient.

EXAMPLES.

Divide 612 Pounds among 100 Men.

1|00)6|12(6 Pounds each.
6

The 12 which is cut from the Dividend is the Remains.

Divide

Divide 3742876 by 67400

```
674|00)37428|76(55
      3370
      ----
       3728
       3370
       ----
        35876   Remains.

                       67400 Divisor.
                          55 Quotient.
                       -----
                      337000
                     337000
                     -------
                     3707000 Product.
                       35876 Remains.
                     -------
                     3742876 Proof.
```

Divide 37100 by 55.

```
55)37100(674              674 Quotient.
   330                     55 Divisor.
   ---                    ---
    410                  3370
    385                 3370
    ---                 -----
     250                37070 Product.
     220                   30 Remains.
     ---                -----
      30 Remains.       37100 Proof.
```

If

If there be never ſo many Cyphers in the Dividend, and none to the right Hand of your Diviſor, you muſt cut none of; as in this Example.

There are four Cannon, as *A. B. C.* and *D.* appointed at a Battery. *A* ſpends at a Shot 9 Pounds of Powder, *B* 5 Pounds, *C* 4 Pounds, and *D* 2 Pounds; and there is 1400 Pounds of Powder to be ſpent by theſe four Pieces, how many Shots can they make each, and how much Powder did each conſume?

A. ſpends 9 at a Shot.
B. ſpends 5 at a Shot.
C, ſpends 4 at a Shot.
D. ſpends 2 at a Shot.

The Diviſor 20

2|0) 140|0 (70 Shots each.
14
0

When they were all once diſcharged, they had conſumed 20 Pounds of Powder, then it is plain dividing the Quantity by what they conſume at one Shot, gives in the Quotient how many Shots it will ſupply them with; then multiplying the Number of Shots by what each ſpends at a Shot, gives the Quantity of what each ſpends in the whole.

70 multiplied by 9 gives 630, what *A.* ſpends.
70 multiplied by 5 gives 350, what *B.* ſpends.
70 multiplied by 4 gives 280, what *C.* ſpends.
70 multiplied by 2 gives 140, what *D.* ſpends.

1400 Proof.

Diviſion of Factors is performed in the following Manner.

Divide 43904 by 56

7|43904

8|6272

784

Divide 78894 by 81

9|78894

9|8766
974

In the firſt of the Examples, I divide firſt by 7, and place the Quotient under the Dividend; then I divide that Quotient by 8, and find the next Quotient 784, the ſame as if I had divided the firſt Dividend by 56, becauſe 7 times 8 is 56.

In the ſecond Example, I divide firſt by 9, and that Quotient by 9 again, and find the laſt Quotient 974; becauſe 9 times 9 is 81.

Diviſion

Division of Compound Quantities

May be easily performed by the Help of the Multiplication of Tens, in the following Manner. Let the Question to be resolved be this:

If one Yard cost 7 s. 4 d. how many Yards may I have for 273 l. 10 s. 8 d.?

s. d.		l. s. d.	l. s. d.	
7 : 4	Divis. Un.	36 : 13 : 4	273 : 10 : 8	746
10			256 : 13 : 4	
3 : 13 : 4	Divis. Ts.	3 : 13 : 4	16 : 17 : 4	
10			14 : 13 : 4	
36 : 13 : 4	Divis. H.	7 : 4	2 : 4 : 0	
			2 : 4 : 0	

You see by the precedent Example, that the Divisors change Place every Operation, and gradually descend in a subdecuple Ratio, but the Quotient continues in the same Place; and as the Multiplication of Tens is very easy, those Divisors are easily obtained, and may be found to what Place you please.

If a Piece of Cloth cost 10 l. 16 s. 8 d. how many Yards are in the same, when the Yard is worth 8 s. 4 d. ?

s. d.		l. s. d.	l. s. d.	
8 : 4 Divis. Un.		4 : 3 : 4	10 : 16 : 8	26 Yards.
10			8 : 6 : 8	
4 : 3 : 4 Divis. Ten.		8 : 4	2 : 10 : 0	
			2 : 10 : 0	
			0	

How many Yards can I have for 12 l. 11 s. 5 d. $\frac{1}{2}$, at 7 s. 4 d. $\frac{3}{4}$ *per* Yard ?

	l. s. d.	l. s. d.	
	3 : 13 : 11$\frac{1}{2}$	12 : 11 : 05$\frac{1}{2}$	34 Ys
		11 1 : 10$\frac{1}{2}$	
s. d.			
7 : 4$\frac{3}{4}$ Un. Div.	7 : 4$\frac{3}{4}$	1 : 9 : 7	
10		1 : 9 : 7	
3 : 13 : 11$\frac{1}{2}$ T. Divis.		0 : 0 : 0	

The Division of a Pound Troy.

24 Grains.	make	one Penny Wt.
20 Penny Weight.	make	one Ounce.
12 Ounces.	make	one Pound.

14 Ounces, 12 Penny Weight, make 1 Pound Averdupoise.

Division of Averdupoise Weight.

16	Drams	make	1 Ounce.
16	Ounces	make	1 Pound.
14	Pound	make	1 Stone.
28	Pound	make	$\frac{1}{4}$ Hundred.
56	Pound	make	$\frac{1}{2}$ Hundred.
112	Pound	make	1 Hundred.
5	Hundred	make	1 Hogshead.
10	Hundred	make	1 Pipe.
20	Hundred	make	1 Tun, or Load.

Apothecaries Weight.

24	Grains of Wheat	make	1 Scruple.
3	Scruples	make	1 Dram.
8	Drams	make	1 Ounce.
12	Ounces	make	1 Pound.

Long Measure.

3	Barley Corns	make	1 Inch.
4	Inches	make	1 Palm.
12	Inches	make	1 Foot.
3	Feet	make	1 Yard.
5	Feet	make	1 Geomet. Pace.
6	Feet	make	1 Fathom.
$5\frac{1}{2}$	Yards	make	1 Perch.
40	Perch	make	1 Furlong.
8	Furlongs	make	1 Mile.
3	Miles	make	1 League.

Cloath

Cloth Measure.

4 Nails	make	1 Quarter.
4 Quarters		1 Yard.
5 Quarters		1 Ell English.
3 Quarters		1 Ell Flemish.

Dry Measure.

2 Pints, or Pounds	make	1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
2 Gallons		1 Peck.
4 Pecks		1 Bushel Land Measure.
5 Pecks		1 Bushel Water Measure.
4 Bushels		1 Coomb.
2 Coombs		1 Quarter.
4 Quarters		1 Chalder.
5 Quarters		1 Tun, or Wey.

Time.

60 Seconds	make	1 Minute.
60 Minutes		1 Hour.
24 Hours		1 Day natural.
7 Days		1 Week.
4 Weeks		1 Month.
13 Months 1 Day		1 Year, or 365 Days.

Liquid Measure.

2 Pints	make	1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
8 Gallons		1 Fir. Ale Soap or Her.
9 Gallons		1 Firkin of Beer.
2 Firkins		1 Kilderkin.
2 Kilderkins		1 Barrel.
42 Galllons		1 Tierce.
63 Gallons		1 Hogshead.
2 Hogsheads		1 Pipe, or Butt.
2 Buts, or 252 Gal.		1 Tun.

REDUCTION

TEaches to bring a Number from one Denomination to another, whereby we know how many of one Denomination are equal to so many of another.

Reduction is performed by Multiplication, secondly by Division, thirdly, by Multiplication and Division.

EXAMPLES.

EXAMPLES.

Question 1st. Reduce 24 l. 7 s. 4 d. $\frac{1}{4}$ into Farthings.

```
  24 : 7 : 4¼
  20
 -----
  487 Shillings.
   12
 -----
  978
 487
 -----
 5848 Pence.
    4
 -----
23393 Farthings.
```

I multiply first by 20, and as I multiplied I brought in the 7 Shillings, so that in 24 l. 7 s. I find 487 Shillings; this I multiplied by 12, the Pence in one Shilling, bringing in the 4 Pence, and I find the Pence 5848; this Number I multiply by 4, the Farthings in one Penny, bringing in the Farthing, and it makes 23393 Farthings.

Question 2d. In 742 Yards, how many Quarters and Nails?

```
  742 Yards.
    4
 -----
 2968 Quarters in 742 Yards.
    4
 -----
11872 Nails in 742 Yards.
```

Question

Question 3d. In 21 *C.* 2 *q.* 14 *lb.* how many Ounces?

21 : 2 : 14
4 Quarters in a C.

86
28 Pounds in a Quarter.

692
173

2422
16 Ounces in a Pound.

14532
2422

38752 Ounces the *Answer.*

Question 4th. In 74 Pounds how many Groats?

74
20 Shillings in a Pound.

1480
3 Groats in a Shilling.

4440 *Answer.*

Question 5th. In 4 Baggs of Pepper, weighing each 2 *C.* 3 *q.* 14 *lb.* how many Pounds?

```
   2 : 3 : 14
   4
   --
   11  Quarters in one Bagg.
   28
   --
   92
  23
  ---
  322  Pounds in one Bagg.
    4
 ----
 1288  Pounds in four Baggs.
```

Question 6th. If 5 Eggs be sold for a Penny, how many will buy a Horse worth 12 Pounds?

```
   12  Pounds Value of the Horse.
   20
  ---
  240
   12
 ----
 2880
    5  Eggs for a Penny.
-----
14400  No. of Eggs the Answer.
```

Question 7th. In 74 Hogsheads how many Gallons and Pints?

```
    74
    63  Gallons in a Hogshead.
   ----
   222
  444
  ----
  4662  Gallons.
     8  Pints in a Gallon.
 -----
 37296  Pints the Answer.
```

Question 8th. How many Yards of *Manchester* Tape will pay for an Estate valued at 746 l. 17 s. 7 d. when the Tape is sold for 3 Yards a Penny.

```
    l.    s.   d.
   746 : 17 : 7
    20
  -----
  14937
     12
  -----
  29881
 14937
 ------
 179251
      3  Yards of Tape for a Penny.
 ------
 537753  Yards of Tape the Answer.
```

Question 9th. How many Barley Corns will reach from *York* to *London*, it being 198 Miles?

198	
8	
1584	Furlongs.
40	
63360	Poles.
11	
63360	
63360	
696960	Half Yards.
18	
5575680	
696960	
12545280	Inches.
3	
37635840	B. Corns in 198 Miles.

N. B. All the former Questions is proved by dividing each Product by what you multiplied by, and it is called *Reduction* by *Division*.

Question 10th. In 73919 Farthings, how many Pence, Shillings, and Pounds?

```
          (12)      2|0
4 | 73919 | 18479 | 153|9 | 76 Pounds.
    4       12      14
    --      ---     --
    33      64      13
    32      60      12
    --      --      --
     19      47      19  Shillings.
     16      36
     --      --
      31     119
      28     108
      --     ---
       39     11  Pence.
       36
       --
        3  Farthings.
```

N. B. I divide first by 4, the Farthings in one Penny; the Quotient is the Number of Pence, and the Remains is Farthings; then that Quotient I divide by 12, the Pence in one Shilling, and place it above the Pence, and the Quotient is Shillings, and the Remains Pence; then dividing that Quotient by 20, it gives the Pounds, and the Remains is Shillings; and the Work is done.

Question 11th. A Gentleman left 5 Pounds to be equally divided amongst Eight Persons, what is the Share of each ?

```
     l.
     5
    20
   ———
8)100(12
    8
  ————
    20
    16
  ————
     4
    12
  ————
 8)48(6
   48
  ———
     0
```

N. B. If the Divisor be larger than the Dividend, the Dividend must be brought into the next inferior Denomination, as in this Example; 5 Pounds is brought into Shillings, and that Product is divided by 8, and the Quotient is 12, and 4 remains, which is brought into Pence, and the Pence divided by 8, and the Quotient is 6, so that each Person had 12 s. 6 d. the Answer.

Observe the same in all Questions of the same Kind.

Question 12th. In 37 l. 15 s. how many Sixpences, Fourpences, and Twopences, and of all an equal Number?

```
d.              l.     s.
6               37  :  15
4                20
2               ----
-                755
12 Divisor.       12
               ------
                1510
                755
              ---------
              12)9060(755
                 84
                ----
                 66
                 60
                -----
                  60
                  60
                -----
                  0
```

The Reason I divide by 12, is because I find one Six-pence, one Four-pence, and one Two-pence is Twelve-pence; so that this Question is only to find how many Shillings is in 37 l. 15 s.; for if one have a Shilling, he has as much as one that has a Six-pence, one Four-pence, and one Two pence.

Explanation

Explanation of the Symbols made Use of in this Arithmetick.

$+$ Signifies Addition, as $7 + 4 + 8 = 19$.
$-$ Signifies Substraction, as $18 - 7 = 11$.
$\times$ Signifies Multiplication, as $9 \times 7 = 63$.
$\div$ Signifies Division, as $48 \div 6 = 8$.
$=$ Signifies Equality, as $7 + 8 + 4 = 19$.
: As, or give.
:: Signifies so is, or so will.

If 4 : 6 :: 8 : 12, that is, if 4 give 6, so will 8 give 12.

N. B. If the Sign of Addition be set after or betwixt any Number of Figures, or Quantities, it shews they must be added into one Sum thus; $7 + 8 + 12 + 14$, when added their Sum will be 41. And so of any other.

And if the Sign of Multiplication be placed in the same Order, they must be multiplied into one another, as in this, $8 \times 7 \times 4 \times 6$, the Product will be 1344.

The *Golden Rule*; or, *Rule of Three.*

THIS Rule is the most excellent and useful Rule of all the Rules in Arithmetick.

It shews how by having three Numbers given, to find a fourth, that shall bear the same Proportion to the Third that the Second doth to the First.

The

The greateſt Difficulty lies in ſtating your Queſtion, which that you may do, obſerve the following Rule; remember that your firſt and third Numbers be of one Name, and the fourth will be of the ſame Name as the ſecond. Then × the ſecond and third together, and ÷ that Product by the firſt. The Quotient will be the Anſwer.

EXAMPLES.

Queſtion 1ſt. If 100 Pounds in one Year gain 4 Pounds Intereſt, what will 75 Pounds gain in that Time?

Place the Numbers in the following Manner.

```
      l.      l.     l.
If 100  :  4  :  75
                  4
                 ---
          1|00)3|00(3 Pounds Anſwer.
                3
               ---
                0
```

100 Pounds the firſt Number is a Principal, and ſo is 75 Pounds the third Number, ſo they are both one Name; 4 Pounds the ſecond Number is an Intereſt, and ſo is 3 Pounds, the Number found, and Anſwer to the Queſtion.

Queſtion 2d. If 1 Acre of Land be worth 15 Shillings a Year, what is 40 Acres (of the like Land) worth by the Year? Anſwer 600 Shillings = 30 Pounds.

```
a.    s.     a.    s.
1  :  15  ::  40  :  600
```

Queſtion

Question 3d. Cloth at 5 Shillings *per* Yard, what comes 72 Yards to? *Answer* 18 Pounds.

	y.		s.		y.		s.		l.
For as	1	:	5	::	72	:	360	=	18

Question 4th. Nails at 5 Pence the Hundred, how many for 3 Pence? *Answer* 60. For,

	d.		ns.		d.		n.
As	5	:	100	::	3	:	60.

Question 5th. If one Yard of Cloth cost 6 *s.* what will 74 Yards of the same cost? *Answer* 22 *l.* 4 *s.* For,

	y.		s.		y.		s.		l.		s.
As	1	:	6	::	74	:	444	=	22	:	4.

Question 6th. If 15 Yards of Cloth cost 7 *l.* 10 *s.* what comes 7 Packs to, each Pack containing 8 Parcels, and each Parcel 108 Yards at that Rate? *Answer* 3024 *l.* Before you state Questions of this Nature, reduce them in the following Manner.

$7 \times 8 = 56$ and: $56 \times 108 = 6048$ Yards.

	l.		s.		
And	7	:	10	=	150 Shillings. Then say

ys. s. ys.
If 15 : 150 :: 6048

```
        150
     ------
     302400
     6048
   --------
15)907200(60480 Shillings. The
   90          Answer = 3024 l.
   --
    72
    60
    --
    120
    120
    ---
      0
```

Question 7th. If 4 lb. of Tea cost 1 *l.* 13 *s.* 4*d.* what will 4 Baggs cost, each Bag weighing 2 C. 3 q. 21 lb. at that Rate? When reduced the Stating will stand thus:

	lb	:	d.	::	lb	:	d		l.		s.		d.
If	4	:	400	::	1316	:	131600	=	548	:	6	:	8

N. B. You may contract Questions in this Rule when you can divide the first and third Term by one Divisor, as in this Question, I divide them by 4, and then the Work will stand thus.

	lb	:	d.	::	lb	:	d.		l.		s.		d.
If	1	:	400	::	329	:	131600	=	548	:	6	:	8

as before.

Question

Question 8th. If 108 Pounds in 12 Months gain 6 Pounds Intereſt, how much will it gain in 7 Months? *Anſwer*, 3 *l*. 10 *s*. 0 *d*.

m. : l : m.
For as 12 : 6 :: 7 : 3 *l*. 10 *s*. 0 *d*.

The *Rule of Three Reverſe*

IS that by which we find, from three Numbers given, a fourth, in a reciprocal Proportion, inverted to the Proportion given. Obſerving the Rule given in direct Proportion, it will be needleſs to ſay any thing more of it.

EXAMPLES.

Queſtion 1ſt. If 8 Men do a Piece of Work in 12 Days, how many Men muſt there be to do the ſame in 6 Days? *Anſwer*, 16 Men.

D. : M. : D.
For if 12 : 8 : 6
12

6)96(16 Men the *Anſwer*.
6
36
36
0

In this Queſtion, I conſider that the leſs Time, will require the more Men; ſo I multiply the mid-

dle Term by the greater Extream, and divide that Product by the lefs, and the Quotient is the Anfwer. Obferve this in all the following Queftions.

Queftion 2d. If 8 Men do a Piece of Work in 12 Days, in how many Days will 16 Men do the fame? *Anfwer*, 6 Days.

m. : d. :: m.
For if 8 : 12 :: 16
8

16)96(6 Men the *Anfwer*.
96

0

Queftion 3d. How many Yards of Canvas that is Ell wide, will line 20 Yards of Say three Quarters wide. *Anfwer*, 12.

q. y. q. y.
For if 3 : 20 :: 5 : 12; for the wider the Cloth the lefs Number of Yards it requires. So if I × 20 by 3, and ÷ by 5.

Queftion 4th. How many Foot of Matting of 2 Foot wide, will cover a Floor 20 Foot wide and 24 Foot long? *Anfwer*, 240 Foot.

f.w. : f.l. :: f.w. : f.l.
For if 20 : 24 :: 2 : 240

For the narrower the Matting, the more it will require; fo I multiply 20 by 24, and divide by 2. The Quotient is the *Anfwer*.

Queftion 5th. If (according to the Statute) a right angled Paralelogram of 40 Perches in length, and 4 in

in breadth, make an Acre of Land, how much in length muſt there be, when the breadth is 16 Perches? *Anſwer* 10 Perches.

b. l. b. l.
For if 4 : 40 :: 16 : 10

Queſtion 6th. If 12 Inches in length and 12 in breadth make a ſuperficial Foot, how much muſt there be in length when the breath is 4 Inches? *Anſwer*, 36 Inches long.

i.l. i.b. i.b. i.l.
For if 12 : 12 : :: 4 : 36

Queſtion 7th. If one lend a Friend 640 Pounds 8 Months, what Sum muſt he lend 12 Months to retaliate the former Favour? *Anſwer*, 426l. 13s. 4d.

m. l. m.
For if 8 : 640 :: 12 : 426l. 13s. 4d.

Queſtion 8th. If one lends his Friend 600 Pounds 8 Months, how long muſt he lend 400 Pounds to recompence the former Kindneſs? *Anſwer*, 12 Months.

l. m. l. m.
For if 600 : 8 :: 400 : 12

Queſtion 9th. If I have 1200 lb. Weight carried 36 Miles for 24 Shillings, how many Miles ſhall 1800 lb. Weight be carried for the ſame Sum. *Anſwer*, 24 Miles.

lb. m. lb. m.
For if 1200 : 36 :: 1800 : 24.

Queſtion 10th. If for 24 Shillings, I have 1200 lb. Weight carried 36 Miles, how many Pound Weight

 ſhall

ſhall I have carried 24 Miles for the ſame Money? *Anſwer*, 1800 ℔. Weight.

m. ℔. m. ℔.
For If 36 : 1200 :: 24 : 1800

Queſtion 11th. A General is beſieged in a Town, in which are 1200 Soldiers, with Proviſion for 3 Months; how many Men muſt he diſmiſs, that his Proviſion may ſerve thoſe that remains 8 Months? He muſt keep 450, and diſmiſs 750.

m. s. m. s.
For if 3 : 1200 :: 8 : 450

Queſtion 12th. If a Field will graze 21 Horſes 6 Weeks, how many will it graze 7 Weeks? *Anſwer*, 18 Horſes.

w. h. w. h.
For if 6 : 21 :: 7 : 18

Queſtion 13th. If a Field will graze 21 Horſes for 6 Weeks, how long will it graze 18 Horſes? *Anſwer*, 7 Weeks.

h. w. h. w.
For if 21 : 6 :: 18 : 7

Queſtion 14th. If 136 Maſons build a Fort in 28 Days, to preſerve the Soldiers from the Enemy, but the General would have it built in 8 Days, how many Men muſt be ſet to Work? *Anſwer*, 476 Maſons.

d. m. d. m.
For if 28 : 136 :: 8 : 476

The

The Double Rule of Three; or, Rule compoſed of five Numbers.

I Shall give no Directions about placing the Numbers, more than what has been delivered in the direct and reverſe Rules, only I ſhall make two ſingle Statings, and work the Queſtion as if it had but one Stating, by the help of the Letter X; the uſe of which will be better underſtood by an Example than a great many Words.

Queſtion 1ſt. If 100 l. Principal in 12 Months, gain 6 l. Intereſt, what will 75 l. Principal gain in 9 Months? *Anſwer* 3 l. 7 s. 6.

$$\left.\begin{array}{l} \text{If } 100 : 6 :: 75 : X \\ \text{If } 12 : X : 9 \end{array}\right\} \text{Theſe are Direct.}$$

100 × 12 = 1200 the Diviſor.
And 75 × 6 = 450 × 9 = 4050 the Dividend.
And 4050 ÷ by 1200 gives 3l. 7s. 6d. the *Anſwer*.

N. B. The X always ſupplies the fourth Place in the firſt Stating, and the ſecond Place in the ſecond Stating; and is ſuppoſed to be of the ſame Name as the Anſwer is of.

Queſtion 2d. If 100 l. Principal, in 12 Month gain 6 l. in how long Time will 75 l. gain 3 l. 7 s. 6 d.? *Anſwer*, 9 Months.

Reduce 6 l. and 3 l. 7 s. 6 d. into Pence, and the Statings will ſtand thus:

If

If $1440 : 11 :: 810 : X$. This is Direct.
If $100 : X :: 75$ This is Reverſe.
$1440 \times 75 = 108000$ The Diviſor.

And $810 \times 12 = 9720 \times 100 = 972000$ the Dividend. Then $972000 \div 108000$ gives 9 in Quotient the *Anſwer*.

Queſtion 3d. If 100 l. Principal, in 12 Months gain 6 Pounds, what Principal will gain 3 l. 7. 6 d. in 9 Months? *Anſwer*, 75 l.

Reduce the Intereſts, and it will ſtand thus.
If $1440 : 100 :: 810 : X$. This is Direct.
If $12 : X : 9$ This is Reverſe.
Then $1440 \times 9 = 12960$ the Diviſor.

And $810 \times 100 = 81000 \times 12 = 972000$ the Dividend. And $972000 \div 12960$ gives 75 in the Quotient the *Anſwer*. Theſe three Queſtions mutually prove one another.

Queſtion 4th. If the Carriage of 100 lb Weight 30 Miles coſt 1 Shilling, what will the Carriage of 500 lb Weight coſt, being carried 100 Miles?

s.
If $100 : 1 :: 500 : X$. This is Direct.
If $30 : X :: 100$ This is Direct.
Then $100 \times 30 = 3000$ The Diviſor.
And $500 \times 1 = 500 \times 100 = 50000$ the Divid.
And $50000 \div 3000 = 16$ s. 8 d. the *Anſwer*.

Queſtion 5th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Quarters will ſuffice an Army of 11232 Soldiers 56 Days?

If

If 936 : 351 :: 11232 : X. This is Direct.
If 168 : X :: 56 This is Direct.
Then 936 × 168 = 157248 the Divisor.
And 11232 × 351 = 3942432 × 56 = 220776192 the Dividend.
And 220776192 ÷ 157248 gives 1404 Quarters the *Answer*.

Question 6th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Soldiers will 1404 Quarters suffice 56 Days?
If 351 : 936 :: 1404 : X This is Direct.
If 168 : X :: 56 This is Reverse.
Then 351 × 56 = 19656 The Divisor.
And 1404 × 936 = 1314144 × 168 = 220776192 the Dividend.
And 220776192 ÷ 19656 gives 11232 the Soldiers it will suffice the *Answer*.

Question 7th. If a Regiment of 936 Soldiers eat 351 Quarters of Wheat in 168 Days, how many Days will 1404 Quarters suffice an Army of 11232 Soldiers?
If 351 : 168 :: 1404 : X This is Direct.
If 936 : X :: 11232 This is Reverse.
Then 11232 × 351 = 3942432 The Divisor.
And 1404 × 168 = 235872 × 936 = 220776192 the Dividend.
And 220776192 ÷ 3942432 gives 56 Days the *Answer*.

Question

Question 8th. If 8 Men in 7 Days mow 40 Acres, how many Acres will 24 Men mow in 28 Days?

If 8 : 40 :: 24 : X This is Direct.
If 7 : X :: 28 This is Direct.
Then 8×7=56 the Divisor.
And 24×40=960×28=26880 the Dividend.
And 26880÷56 gives 480 Acres the *Answer.*

Question 9th. If 8 Men in 7 Days mow 40 Acres, in how many Days will 480 Acres be mowed by 24 Men?

If 8 : 7 :: 24 : X This is Reverse.
If 40 : X :: 480 This is Direct.
Then 24×40=960 The Divisor.
And 480×8=3840×7=26880 The Dividend.
Then 26880÷960 gives 28 The *Answer.*

Question 10th. If 8 Men in 7 Days mow 40 Acres, how many Men must there be to mow 480 Acres in 28 Days?

If 7 : 8 :: 28 : X This is Reverse.
If 40 : X :: 480 This is Direct.
Then 28×40=1120 The Divisor.
And 480×8=3840×7=26880 The Dividend.
Then 26880÷1120=24 Men the *Answer.*

Question 11th. If 40 Shillings be the Wages of 8 Men for 5 Days, what will be the Wages of 32 Men for 24 Days?

If 8 : 40 :: 32 : X This is Direct.
If 5 : X :: 24 This is Direct.
Then 5×8=40 the Divisor.
And 32×40=1280×24=30720 The Dividend.
Then 30720÷40=768 Shillings=38l. 8s. The *Answer.*

Question

Question 12th. If 14 Horſes eat 56 Buſhels of Provender in 16 Days, how many Buſhels will 20 Horſes eat in 24 Days?

If 14 : 56 :: 20 : X This is Direct.
If 16 : X :: 24 This is Direct.
Then 16×14=224 the Diviſor.
And 56×20=1120×24=26880 The Dividend.
Then 26880÷224 gives 120 Buſhels the *Anſwer*.

Question 13th. If 8 Cannons in one Day ſpend 48 Barrels of Powder, how many Barrels will 24 Cannons ſpend in 12 Days?

If 8 : 48 :: 24 : X This is Direct.
If 1 : X :: 12 This is Direct.
Then 1×1=8 The Diviſor.
And 24×12=288×48=13824 The Dividend.
Then 13824÷8 gives 1728 Barrels the *Anſwer*.

Question 14th. If in a Family conſiſting of 7 Perſons, there are drank out 36 Gallons of Beer in 12 Days, how many Gallons will there be drank out in 8 Days by another Family, conſiſting of 14 Perſons?

If 7 : 36 :: 14 : X This is Direct.
If 12 : X :: 8 This is Direct.
Then this 12×7=84 The Diviſor.
Then 14×8=112×36=4032 The Dividend.
And 4032÷84 gives 48 Gallons the *Anſwer*.

Question 15th. If, when the Buſhel of Wheat coſts 3s. 4d. the Penny Loaf weighs 12 Ounces, what will a Loaf worth 9 Pence weigh, when Wheat is 10 Shillings the Buſhel?

If 40 : 12 :: 120 : X This is Reverse.
If 1 : X :: 9 This is Direct.
Then 120×1=120 The Divisor.
And 40×12=480×9=4320 The Dividend.
And 4320÷120=36 Ounces the *Answer*.

Question 16th. A Man put out 75 l. to receive Interest for the same, and when it had continued 9 Months, he received (for Principal and Interest) 78 l. 7 s. 6 d.; now at what Rate per Cent. *per Annum* did he receive Interest?

If 75 : 810 :: 100 : X This is Direct.
If 9 : X : 12 This is Direct.
Then 75×9=675 The Divisor.
And 810×100=81000×12=972000 Dividend.
Then 972000÷675 gives 1440 Pence=6 Pound the *Answer*.

Question 17th. If a Captain set 300 Pioneers to work, and they in 8 Hours cast a Trench of 200 Rods, how many Pioneers will be able, with a like Trench in 3 Hours, to intrench a Camp 34000 Rods?

If 200 : 300 :: 3400 : X This is Direct.
If 8 : X :: 3 This is Reverse.
This 200×3=600 The Divisor.
And 3400×300=1020000×8=8160000 The Dividend.
Then 8160000 ÷ 600 gives 13600 Pioneers the *Answer*.

Question 18th. If 20 Dogs for 30 Groats go 40 Weeks to Grass, how many Hounds for 60 Crowns may winter in that Place? *N. B.* Twelve Weeks is a Winter.

C. D. C.
If 2 : 20 :: 60 : X This is Direct.
W. D. W.
If 40 : X :: 12 This is Reverse.
Then $12 \times 2 = 24$ The Divisor.
And $20 \times 60 = 1200 \times 40 = 48000$ The Dividend.
Then $48000 \div 24$ gives 2000 Dogs the *Answer.*

Question 19th. If 35 Ells of Vienna make 24 at Lyons, and 3 Ells at Lyons make 5 Ells at Antwerp, and 100 Ells at Antwerp 125 at Franckfort, how many Ells at Franckfort make 42 at Vienna? *Answer* 60 Ells.

$$\left.\begin{array}{l} \text{If } 100 : 125 :: 5 : X \\ \text{If } 3 : X :: 24 : X \\ \text{If } 35 : X :: 42 \end{array}\right\} \text{These are all Direct.}$$

Then $100 \times 3 = 300 \times 35 = 10500$ The Divisor.
And $125 \times 5 = 625 \times 24 = 15000 \times 42 = 630000$ The Dividend.
Then $630000 \div 10500$ gives 60 Ells the *Answer.*

Notation of Vulgar Fractions.

A Fraction is properly an Unit divided into Parts, and is noted by two Numbers thus $\frac{7}{12}$, the lower denoteth the Number of Parts the Unit is divided into, and is called the Denominator; the upper ſhews how many of theſe Parts are ſignified, and is called the Numerator.

As if the Integer be one Shilling broken into Twelve Parts, then this Fraction $\frac{7}{12}$, is in Value 7 Pence, and is read ſeven Twelfths.

Fractions are of three Kinds, *viz.*

This $\frac{7}{12}$ is a ſimple Fraction; the Numerator is the leaſt. This $\frac{1}{2}$ of $\frac{3}{4}$ is a Compound Fraction, or a Part of a Part. This $\frac{5}{4}$ is an improper Fraction; the Numerator is the largeſt.

N. B. If the Numerator be equal to, or greater than the Denominator, it is an improper Fraction. Thus $\frac{4}{4}=1$ and $\frac{7}{4}=1$ and $\frac{3}{4}$.

Reduction of Vulgar Fractions

TEacheth to bring Fractions of different Denominators, into one common Denominator. To bring a Compound Fraction to a Simple one; and to expreſs a Number like an improper Fraction.

To bring Simple Fraction into one common Denominator. *Rule*, × all the Denominators into one

another

another for a common Denominator; and every Numerator into all the Denominators, but its own, for new Numerators.

EXAMPLES.

Reduce $\frac{2}{3}$. $\frac{3}{4}$. $\frac{5}{6}$. and $\frac{1}{2}$ into a Common Numerator.

Firft $3\times4\times6\times2=144$ the Common Denominator.

Then $2\times4\times6\times2=96$ New Numerator $=\frac{2}{3}$.
And $3\times3\times6\times2=108$ New Numerator $=\frac{3}{4}$.
And $5\times4\times3\times2=120$ New Numerator $=\frac{5}{6}$.
And $1\times6\times4\times3=72$ New Numerator $=\frac{1}{2}$.

The New Fractions will ftand thus:

$\frac{96}{144}$ $\frac{108}{144}$ $\frac{120}{144}$ $\frac{72}{144}$ New Numerators. Common Denominator.

N. B. If I had divided my Common Denominator by the Denominator of each Fraction, and $\times$ that Quotient by the Numerator of the Fraction, the Product would be a new Numerator, as before.

By either of thefe Ways you may reduce Simple Fractions to one Common Denominator.

To reduce Compound Fractions to Simple ones. *Rule*, $\times$ all the Numerators together for a new one, and $\times$ all the Denominators together for a new one.

EXAMPLE.

EXAMPLE.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ to a Simple Fraction.
First $2\times3\times1=6$ the Numerator.
And $3\times4\times2=24$ the Denominator.
Then the Simple Fraction will be $\frac{6}{24}$.

To reduce a mixt Number to an improper Fraction. *Rule*, $\times$ the whole Number by the Denominator of the Fraction, and to the Product add the Numerator of the Fraction; and under this Sum place the Denominator of the Fraction, and it is done.

EXAMPLE.

Reduce $4\frac{3}{4}$ to an improper Fraction.
$4\times4=16+3=19$. Then the Fraction is $\frac{19}{4}$.

Express 7 like an improper Fraction.
Then it is $\frac{7}{1}$, or $\frac{14}{2}$, or $\frac{21}{3}$, or $\frac{63}{9}$, &c. Any of these Fractions are in Value 7.

To reduce an improper Fraction to a mixt Number. *Rule*, Divide the Numerator by the Denominator, and the Remainder place over the Divisor, and its done.

Thus $\frac{19}{4} = 4\frac{3}{4}$.

EXAMPLE.

To bring a Fraction to its lowest Terms. *Rule*, Divide the Denominator by the Numerator, and if any thing remain, divide your last Divisor by it, so every Divisor, if there is a Remainder, becomes the Dividend, and the Remainder your Divisor; and the last Divisor, if there be no Remainder, is the Number

Number that will reduce the Fraction to its lowest Term. If there is no such Divisor, the Fraction is in its lowest Terms.

EXAMPLE.

Reduce $\frac{1008}{1512}$ into its lowest Term. Place it thus:

```
1008)1512(1
     1008
     ————
      504)1008(2
          1008
          ————
             0
```

Now 504, which was my first Remainder and last Divisor, will divide both the Numerator and Denominator of the given Fraction, and so reduce it to $\frac{2}{3} = \frac{1008}{1512}$.

Reduce $\frac{2}{3}$ and $4\ \frac{3}{7}$ and $\frac{1}{2}$ of $\frac{3}{4}$, to a Common Denominator.

The mixt Number reduced to an improper Fraction, is $\frac{31}{7}$.

The Compound Fraction reduced to a Simple one, is $\frac{3}{8}$.

Then for Reduction, they will stand in the following Manner:

$\frac{2}{3}$. $\frac{31}{7}$ and $\frac{3}{8}$.

Then $3 \times 7 \times 8 = 168$ the Common Denominator.
And $2 \times 7 \times 8 = 112$ the New Numerator $= \frac{2}{3}$.
And $31 \times 3 \times 8 = 744$ the New Numerator $= \frac{31}{7}$.
And $3 \times 7 \times 3 = 63$ the New Numerator $= \frac{3}{8}$.
And they will stand thus $\frac{112}{168}$ $\frac{744}{168}$ $\frac{63}{168}$.

To reduce a Fraction of one Denomination to another.

What part of a Pound is $\frac{1}{4}$ of a Penny; it will be this Compound Fraction, $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}=\frac{1}{960}$

To bring a Fraction from a greater to a less Denomination. *Rule*, multiply the Numerator by the Parts contained betwixt it, and what you would reduce it to.

EXAMPLE.

Reduce $\frac{2}{3}$ of a Pound to the Fraction of a Penny

$2\times12\times20=480$ then it is 480

To find the Value of any Fraction in the known Parts of Coin, Weight, Measure, &c. *Rule*. Multiply the Numerator of the Fraction by the next inferior Denomination, that is equal to a Unit of the same Denomination with the Fraction, and divide that Product by the Denominator of the given Fraction, and the Quotient is its Value, in the Parts you multiplied by; and if there be a Remainder after your Division, multiply by the next Inferior, and divide as before, and so continue to do till you can bring it no lower.

What

What is the Value of $\frac{133}{480}$ of a Pound Sterling?
Answer 5 s. 6 d. $\frac{1}{2}$

```
    133
     20
   ------
480)2660(5
    2400
    ----
     260
      12
    ----
     520
    260
482)3120(6
    2880
    ----
     240
       4
    ----
480)960(2
    960
    ---
      0
```

Addition of Vulgar Fractions.

R U L E.

BRING the Fractions to be added into one Denominator by the Rules in Reduction; then add their Numerators, and under the Sum put the Common Denominator, and it is done.

EXAMPLE.

Add $\frac{2}{3}$ $\frac{3}{4}$ and $\frac{5}{6}$ of a Pound into one Sum.
$3\times4\times6=72$, the Common Denominator.
The Numerators will be found 48, 54, and 60.
Then $48+54+60=162$. The Sum is $\frac{162}{72}$.

Substraction of Vulgar Fractions.

RULE.

BRING the Fractions to one Denominator, and subtract the Numerators one from another, and under place the Common Denominator, and it is done.

EXAMPLE.

From $\frac{3}{4}$ take $\frac{1}{8}$. First $8\times4=32$ Common Denominator.

And $3\times8=24$ New Numerator. And $2\times4=8$. Then $\frac{24}{32}-\frac{8}{32}=\frac{16}{32}$.

Multiplication of Vulgar Fractions.

RULE.

REduce mixt Numbers into improper Fractions, express whole Numbers like Fractions, and bring Compound Fractions into Simple. Then multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator; which Numerator and Denominator is the Product sought.

EXAMPLE.

Multiply 2s. 6d. by 2s. 6d. as 2s. 6d.$=\frac{1}{8}$.
Then $\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}=$0l. 0s. 3d.$\frac{3}{4}$. And $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Division

Division of Vulgar Fractions.

REduce the Fractions by the Rule in Multiplication, then invert your Divisor, and multiply then as in Multiplication, and the Product will be your Quotient.

EXAMPLE.

Divide $\frac{8}{117}$ by $\frac{4}{13}$, the Divisor invert, it will stand thus: $\frac{13}{4})\frac{8}{117}(\frac{104}{468}$, the Quotient; for $13\times8=104$. And $117\times4=468$.

Divide $\frac{36}{114}$ by $\frac{6}{19}$. Invert the Divisor, and it will stand thus: $\frac{19}{6})\frac{36}{114}(\frac{684}{684}=1$.

Arithmetical Progression

IS when a Rank of Numbers above two increase or decrease by the continual Addition or Subtraction of some equal Number. So 2, 7, 12, 17, 22, 27, 32, and 36, 32, 28, 24, 20, 16, 12, 8, 4, are two Ranks of Numbers in Arithmetical Progression; the first increasing by the continual Addition of 5, and the second decreasing by the continual Subtraction of 4, and so of any other. In this Progression five Things are to be considered, viz.

1st. The first Term.
2d. The last Term.
3d. The Number of Terms.
4th. The common Difference.
5th. The Sum of all the Terms.

Any of these being given, the other may be found, and admits of 20 Varieties. See *Oughtred's* Key of the Mathematicks, Page 85.

Theorem 1st. The last Term is always = to the first, and so many common Differences as there are Number of Terms less by one, as in this Progression, 4, 9, 14, 19, 24, 29. Here the Number of Terms is 6, and the common Difference is 5, and the first Term is 4. Then $6-1=5$ the Number of Terms less one. And $5\times5=25+4=29$ the last Term.

Theorem 2d. If the first Term is added to the last, and that Sum $\times$ by the Number of Terms, the Product will be double the Sum of the whole Progression.

As in the last, $4+29=33$.
And $33\times6=198$ double the Sum of the whole,
Then $198\div2=99$ the Sum of the Progression.

Question 1st. A Merchant bought 60 Bales of Holland, and agreed to pay for the first Bale 4l. for the second 7l. so increasing the Price of every Bale by 3l. what did he pay for the last Bale, and what did the whole cost?

Here is given 4 the first Term, and 3 the common Difference, and 60 the Number of Terms. Then $59\times3=177$; to this add the first Term, and it will be $177+4=181$ l. the Price of the last Bale. Then $181+4=185$, and $185\times60=11100$, this $\div$ $2=5550$ l. the Price of the whole.

If

If 100 Eggs are placed in a right Line, a Yard diſtant from one another, and the firſt a Yard diſtant from a Baſket, how far muſt one go before he brings the Eggs one by one into the Baſket.

N. B. They muſt begin at the Baſket.

2	Firſt Term.
200	Laſt Term.
202	The Sum of the firſt and laſt Term.
100	The Numbers of Terms.
20200	Double the Yards gone. The half is
10100	Yards gone in the whole = To 5 Miles and 1300 Yards.

Theorem 3d. If double the Sum be divided by the Sum of the firſt and laſt Terms, the Quotient will be the Number of Terms.

Theorem 4th. If the firſt Term be ſubtracted from the laſt, and the Remainder divided by the Number of Terms leſs by one; the Quotient will be the common Difference.

A Gentleman ordered his Steward to diſburſe 1700 Pounds amongſt his Tenants, that had ſuffered by the Contagion amongſt the Cattle, and bring him an Account of the Number of Sufferers, and how much each received of his Bounty; but the Steward loſt the Account, and only remembers that the leaſt Sufferer had 3l. and the greateſt 133l. and that the Difference was equal betwixt every two next, How many

many Tenants received of the Bounty, and what was the Difference?

The last Term 133
The first Term 3

Their Sum is 136 the Divisor.
And $1700 \times 2 = 3400$ the Dividend.
Then $3400 \div 136 = 25$ the Number of Tenants;
And $133 - 3 = 130$l. and this divided by 24, gives 5l. 8s. 4d. the Difference betwixt every two next. So that the least received 3l. the next 8l. 8s. 4d. the third 13l. 16s. 8d, &c.

Geometrical Progression

IS when Numbers increase by a like Proportion, as 4, 8, 16, 32, 64, 128, are in Geometrical Progression, for the second contains the first as many Times as the third contains the second, and so the fourth the third, &c.

In this Progression, the same Things are to be considered as in Arithmetical Progression; as the first Term, the last Term, the Number of Terms, the Ratio, or common Excess; and the Sum of all the Terms.

If over a Series of Terms in Geometrical Progression, be set for Indices a Series of Terms in Arithmetical Progression, to every four Numbers taken in Arithmetical Progression, will answer four Numbers geometrically proportional.

Examples.

EXAMPLES.

Indices	0.	1.	2.	3.	4.	5.
Terms	4.	8.	16.	32.	64.	128.

These Indices, or Exponents, beginning with a Cypher, and placed over any Series in Geometrical Progression, shews the Distance of any Term from the first.

Theorem 1st. In any Geometrical Progression, if you add the Exponents of any two Terms, and multiply the Terms together, and divide the Product by the first Term, the Quotient will be the Term answering the Sum of the Indices you added.

EXAMPLES.

0.	1.	2.	3.	4.	5.	6.
4.	8.	16.	32.	64.	128.	256.

In this Progression, if the third and fourth Terms be $\times$ together, and the Product divided by the first, the Quotient will be the seventh Term, and so of any other.

That is, $64 \times 32 = 2048 \div 4 = 512$ the seventh Term.

Theorem 2d. If the last Term be $\times$ by the Ratio, and from the Product subtract the first Term, and divide the Remainder by one less than the Ratio, the Quotient will be the Sum of the whole Progression.

A Merchant has 12 Yards of Velvet, which he values at 1l. 4s. per Yard; but he sells it for 1s. the first Yard, 2s. the second Yard, 4s. the third Yard, &c. doubling the Price of each Yard to the twelfth and last Yard. The Question is, what does he gain or lose?

Indices	0.	1.	2.	3.	4.	5.	6.
Terms	1.	2.	4.	8.	16.	32.	64.

Then $64 \times 32 = 2048$ Price of the last Yard.

For the Indices $5+6=11$ the Distance of that Term from the first.

And $2048 \times 2 = 4096 - 1 = 4095$ Shillings; the Value of the 12 Yards at the Price agreed upon = 204l. 15s. which is 17l. 1s. 3d. *per* Yard; so that 17l. 1s. 3d. − 1l. 4s. 0d. = 15 : 17 : 3 what the Merchant gains *per* Yard above the Value.

Suppose an Estate of 72 l. *per* Annum was sold on these Conditions; the first of *January* the Buyer is to pay 3 s. the first of *February* 9 s. the first of *March* 27s. and so on in Geometrical Proportion, to the last and 12th Payment on the first of *December*. The Question is, to know what the 12th Payment was, and what the Estate came to?

Indices	0.	1.	2.	3.	4.	5.	6.
Terms	3.	9.	27.	81.	243.	729.	2187.

Then $2187 \times 729 = 1594323$, this $\div$ by three gives 531441 = the last Payment in Shillings.

Then $531441 \times 3 = 1594323 - 3 = 1594320$.

This $1594320 \div 2 = 797160$ Shillings the Estate cost = 39858 Pounds the *Answer*.

If

If a Horse, having 4 Shoes, and 6 Nails in each Shoe, is sold on this Condition, that the first Nail is to be a Halfpenny, the second a Penny, the third Two-pence, &c. doubling every Nail; how much would the Price of the Horse come to?

Indices	0.	1.	2.	3.	4	5.	6.	7.
Terms	1.	2.	4.	8.	16.	32.	64.	128.

First, $128 \times 128 = 16384$ Price of the 15th Nail.
And, $16384 \times 64 = 1048576$ Price of the 21st Nail.
Then, $1048576 \times 8 = 8388608$ Halfpence the Price of the 24th Nail.
And, $8388608 \times 2 = 16777216 - 1 = 16777215$ The Price of the Horse in Halfpence, equal to 34952l. 10s. 7d. ½.

Variation of Quantities

SHEWS, how many several Ways any Number of Qualities may be changed or varied, with respect to their Places.

EXAMPLE.

How many Changes may be rung on 8 Bells?
Rule. × 1, 2, 3, 4, 5, 6, 7, 8, into one another, and the last Product is the *Answer*.

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ The *Answer*, and so many Changes may be rung on 8 Bells.

How many different Positions may a Captain order 12 Men in?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 =$ 479001600 The *Answer*, and so many different Positions may they be placed in with regard to one another; which is almost incredible.

Notation of Decimal Fractions.

A Decimal Fraction is such, whose Denominator is not expressed, but understood; and is a Unit with as many Cyphers annexed, as there is Places in the Numerator.

So $\frac{5}{10}$ will be expressed thus, ,5; and $\frac{25}{100}$ thus, ,25; and $\frac{425}{1000}$ thus, ,425; and this Point, or Comma is prefixed, to distinguish them from an Integer.

N. B. A Cypher placed to the left Hand of an Integer, and to the right Hand of a Decimal, neither increaseth nor decreaseth the Value, as 4. 04. 004. These being Integers, do still retain the same Value.

And ,7. ,70. ,700. these being Decimals, do retain the same Value, every one being ,7 Parts, when the Integer is divided into 10 Parts.

But Cyphers placed to the right Hand of an Integer, increaseth its Value; and to the left Hand of Decimal, decreaseth its Value.

As ,5 is five Tenth Parts.
,05 is five Hundred Parts.
,005 is five Thousand Parts.
,0005 is five Ten thousand Parts.

By this you may observe, Decimal Fractions do decrease in a Ten-fold Proportion to the right Hand.

Reduction

Reduction of Decimals.

BY this we find the Decimal, equal in Value to any Vulgar Fraction given; and on the contrary, find the Value of a Decimal Fraction in the known Parts of Coin, Weight, Measure, &c.

To bring a Vulgar Fraction to a Decimal. *Rule*, Add as many Cyphers as you please to the Numerator of the given Fraction; and then divide by the Denominator, and the Quotient is the Decimal required. Distinguish the Cyphers you add with a Comma.

EXAMPLES.

Reduce $\frac{3}{4}$ to a Decimal Fraction.

```
4)3,00(,75 The Decimal.
  28
  --
   20
   20
```

Reduce $\frac{1}{12}$ to a Decimal Fraction.

```
12)1,0000(,0833
    96
    --
     40
     36
     --
      40
      36
      --
       4
```


N. B.

N. B. In the above Example, I first say how many Times 12 the Denominator, in 1 the Numerator; and then I make the Comma in the Quotient to distinguish it, and then I say how many times 12 in 10, and I place a Cypher after the Comma; and so you must take your Cyphers one by one, placing Cyphers in your Quotient, if you cannot have the Divisor in your Dividend: And if you annex a thousand Cyphers, there would be the same Remainder, and constantly 3's in your Quotient; and they are called Repetends, or repeating Numbers. But if we bring Decimals to 5 or 6 Places, it will be exact enough in most Cases, and the Remainder will be of no Value.

Reduce Nine Pence into a Decimal, a Pound being the Integer, 240 Pence being a Pound, it will be $\frac{9}{240}$; then add Cyphers to the Numerator, and divide as before.

```
240)9,0000(,0375
    7 20
    ----
    1 800
    1 680
    -----
     1200
     1200
     ----
       0
```

Reduce 15 Shillings into the Decimal of a Pound.

20)15,00(,75 = 15 Shillings.
140
———
100
100
———
0

The Decimal of any Number of Shillings is found by halving them, and placing a Comma before the Half.

What is the Decimal of 4s. 6d. a Pound the Integer?

4s. 6d.
12
———
54

240)54,000(,225=4 Shillings and 6 Pence.
480
———
600
480
———
1200
1200
———
0

What

What is the Decimal of 12 Penny Weight, a Pound Troy being the Integer? 240 Penny Weight is one Pound. Then it will be $\frac{12}{240}$.

240)12,00(,05=12 Penny Weight.
1200
——
0

What is the Decimal of one Inch, a Foot being the Integer?

12(1,0000(,0833
96
——
40
36
——
40
36
——
0

In this Manner is found the Decimal of any Vulgar Fraction whatsoever. I shall omit placing the Tables, because of the easy finding the Decimal by the Rule delivered.

To find the Value of any Decimal Fraction in the known Parts of Coin, Weight, Measure, Time, Motion, &c.

Rule. Multiply the Decimal given by the Number of Parts of the next inferior Denomination, cutting off as many Figures from the Product, as there is in the Decimal given, and the Remainder, (if any) multiply by the Parts of the next Inferior, cutting off as before. Thus must you do, till the Decimal is brought into its least Parts. The Value is what you cut off to the left Hand.

Example.

EXAMPLE.

What is the Value of ,875 of a Pound Sterling?
20 Shillings in a Pound.

17,500
12

1000
500

6,000

So that the Value is 17 s. and 6 d.

What's the Value of ,69687 of a Pound Sterling?

,696875
20

Shillings 13,937500
12

1875000
937500

Pence 11,250000
4

Farthings 1,000000

What

What is the Value of ,72065 of a Pound Sterling?

,72065
20

Shillings 14,41300
12

82600
41300

Pence 4,95600
4

Farthings 3,82400 the Remain less than a Farthing, and may be rejected.

What is the Value of ,6725 of a Hundred Weight?

,6725
4

Quarters 2,6900
28 Pounds in a Quarter.

55200
13800

Pounds 19,3200
16 Ounces in a Pound.

19200
3200

Ounces 5,1200

It

Addition of Decimals

IS not much different from Addition of Integers; only mind to keep Units under Units in Integers, and Tenths under Tenths in Decimal Parts.

EXAMPLE.

Add ,7435 of a Pound to ,75 of a Pound. Place them in the following Manner.

,7435
75
1,4935

N. B. When you have added your Decimals together, ſo many muſt be cut off with a Comma, as that Decimal Number conſiſts of, which in your Example contains the moſt Places. The reſt, (if any are Integers.

£.	£.
,374	42,748
,067	61,145
,5746	37,567
,35	27,915
Sum 1,3656	Sum 169,370

But if the Numbers given to be added are not all of the ſame Denominations, they muſt be brought into Fractions of like Denominations, by finding the Value of the leaſt, and then bringing it by Reduction, to be of the ſame Denomination with the greater.

EXAMPLE.

Add ,375 of a Pound to ,75 of a Shilling.

The ,75 of a Shilling will be found ,0375 of a Pound.

And the Addition will ſtand as follows,

	,375
	,0375
The Sum	,4125

Subtraction of Decimals

DIffers nothing from Subtraction of whole Numbers; only in placing them, mind to place Units under Units in Integers; and Tenths under Tenths, in Decimal Parts.

EXAMPLES.

Subtract ,6725 from ,8765. Place them thus.

From	,8765
Take	,6725
Remains	,2040

So if from	27,625
You ſubtract	8,875
There will remain	18,750

If the Decimal Parts in either Number, have fewer Places than the other, the Vacancy must be supplied, by annexing so many Cyphers as will make them equal, or suppose them annexed.

EXAMPLE.

From	274,4500	From	274,45
Take	89,7425	Take	89,7425
Remains	184,7075	Excess	184,7075

But if your Numbers given to be subtracted, are not of the same Denomination, you must, as in Addition, bring them into one Denomination.

EXAMPLE

Subtract ,375 of a Shilling, from ,25 of a Pound, It will stand thus:

	£.	s. d.
	,25	
	01875	
Remains	,23125 =	4 $7\frac{1}{2}$.

Or if you bring the Decimal of a Pound to that of a Shilling, the Work will stand thus.

From	5,000	
Take	,375	
		= s. d.
	4,625	= 4 $7\frac{1}{2}$.

Multiplication of Decimals.

IS the fame as Multiplication of whole Numbers, only mind to cut off with a Comma from the Product, as many Places as you have Decimals in both your Multiplicand and Multiplier; and if there is not fo many, add Cyphers to the left Hand of the Product.

EXAMPLE.

Multiply	,375	
By	2,5	
	1875	
	750	
	,9375	The Product.

Multiply	,652	
By	,25	
	3260	
	1304	
	,16300	The Product.

If a Board is 7 Foot 9 Inches long, and 4 Foot 6 Inches broad, what is the fuperficial Content?

The Inches being reduced to the Decimal of a Foot, the Work will ftand in the following Manner;

F.

```
       F.
      7.75 Length.
       4,5 Broad.
    ---------
      3875
     3100
    ---------
    34,875
```

The superficial Content $=$ F. : I. 34 : 10 $\frac{1}{2}$

By Crofs Multiplication, the Work will ftand thus:

```
F.  I.
 7 : 9
     4 : 6
----------
 3 : 10 : 6
31 :  0
----------
34 : 10 : 6
```
The Anfwer as before.

Multiply 2 s. 6d. by 2 s. 6d. a Pound being the Integer.

Two Shillings and fix Pence being reduced to the Decimal of a Pound, the Work will ftand thus:

```
   ,125
   ,125
  ------
    625
   250
  125
 --------
 ,015625=0l. 0s. 3d. ¾
```

This

This Question performed by Vulgar Fractions, 2 s. 6 d. is $\frac{1}{8}$ of a Pound. Then $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$, which valued by the Rule given in Vulgar Fractions, is 0 l. 0 s. 3 d. $\frac{3}{4}$ as before.

Multiply 2 s. 6 d. by 2 s. 6 d. a Shilling the Integer.

Six Pence being reduced to the Decimal of a Shilling, the Work will stand thus:

```
  2,5
  2,5
 -----
  125
  50
 -----
 6,25 = 6 s. 3 d.
```

Multiply 4 l. 12 s. 6 d. by 5 l. 15 s. 6 d.
Decimal answering 12 s. 6 d. is ,625.
Decimal answering 15. 6. is ,775.

```
Then multiply   4,625
           By   5,775
              -------
                23125
               32375
              32375
             23125
             ---------
             26,709375 = 26 l. 14 s. 2 d. 1/4.
```

Here follows a Method to contract your Product to what Number of Decimals you please, as in the last Example. I would have the Product consist but of three Places of Decimals. See the Work.

Multiply

Multiply 5,775 by 4,625, and to have but three Places of Decimals.

```
5,775  Multiplicand.
526,4  Multiplier inverted.
------
23101  Mind, one is brought in for the in-
 3465              crease of the Figures.
  115
   28
------
26,709
```

N. B. That you place Unity of your Multiplier under as many Places distant from Unity in your Multiplicand, as you would have Decimals in your Product; then multiply as usual, only mind that you begin your Multiplicand with that Figure that stands over the Figure you multiply by, having respect to the Increase that would come from the following Figures of the Multiplicand, placing every single Product exactly at the right Hand, contrary to the common Way, then add them, and cut off as many as you designed, as you see by the preceding Example. But the common Way is the best.

Division of Decimals

Differs nothing from Division of Integers, either in placing the Numbers, or in the Work itself, the Difficulty being in discovering the true Value of the Quotient, which to do, observe the following Rule.

Rule.

Rule. The Dividend must contain as many Decimals as the Divisor, and the Quotient both together; that is, if there be 7 Decimals in the Dividend, and 3 in the Divisor, there must be 4 in the Quotient, or if there be 7 in the Dividend, and none in the Divisor, there must be 7 in the Quotient, always minding to keep the Dividend equal in Number of Decimals to the Divisor and Quotient.

EXAMPLE.

Divide 1342,332 by 36

```
36)1342,332(37,287
   108
   ---
    262
    252
    ---
     103
      72
     ---
      313
      288
      ---
       252
       252
       ---
         0
```

In the above Example, the Dividend is a mixt Number, and contains 3 Decimals, but the Divisor is an Integer; then by the Rule, there must be 3 Decimals in the Quotient.

EXAMPLE 2d.

Divide 17,3592 by 4,6

4,6)17,2592(3,752
138

345
322

239
230

92
92

0

In this second Example, both the Dividend and Divisor are mixt Numbers, and there is 4 Decimals in the Dividend, and 1 Decimal in the Divisor; then there must be 3 Decimals in the Quotient, to make the Numbers in Divisor and Quotient equal to the Number of Decimals in the Dividend.

EXAMPLE 3d.

Divide 4672,565 by 25,635

25,635)4672,565(182
25635

210906
205080

58265
51270

6995 Remainder.

In this third Example, there is a like Number of Decimals in both the Dividend and Divisor, so the Quotient will be an Integer; and if you add Cyphers to the Remainder, and divide as before, you may bring the Quotient to what Number of Decimals you please.

EXAMPLE 4th.

Divide ,75 by ,0125

0125),7500(60
750
———
00

In this 4th Example, both the Dividend and the Divisor are Decimals; and because I cannot divide the Dividend by the Divisor, I add Cyphers to the Dividend, to make the Number of decimal Places in the Dividend equal to those in the Divisor, and the Quotient is an Integer. And you may observe, that Multiplication of Fractions decreaseth their Value, so Division of Fractions increaseth their Value contrary to the Nature of Integers. This Example is the same as if it were demanded to divide 15 s. by 3d. a Pound being the Integer; For the Decimal of 15s. is ,75, and that of 3d. is ,0125, as you may find by Reduction; and the Quotient will be 60

EXAMPLE 5th.

Divide 1425 by ,6252

,6252)1425,0000(2279
12504
——
17460
12504
——
49560
43764
——
57960
56268
——
1692 Remainder.

You see that before the Division can be made, you must add a competent Number of Cyphers to the Dividend, as I have done, to the Number of 4, which is as many as I have Decimals in my Divisor, and then the Quotient will be Integers; but if you would have Decimals in the Quotient, then you must add as many Decimals to the Dividend, more than there is in the Divisor, as you would have decimal Places in your Quotient; or if you add Cyphers to the Remainder in this Example, and proceed on with your Division, all the Numbers will be Decimals in the Quotient after. As if you would add 7 Decimals to the Dividend at first, or 3 to the Remainder, (for Cyphers are the same as Decimals) the Quotient you will find to be 2279,270, and the Remainder 3960.

EXAMPLE 6th.

Divide, ,358845 by 47

```
47),358845(,007635
    329
    ---
     298
     282
     ---
      164
      141
      ---
      235
      235
      ---
        0
```

In this Example, the Dividend is a Decimal, and the Divisor an Integer, and when the Division is ended, there is but 4 Decimals in the Quotient, and there is 6 in the Dividend, for which Reason, I add 2 Cyphers to the left Hand of my Quotient, to make them equal; as there were none in the Divisor, according to the Rule.

EXAMPLE.

EXAMPLE 7th.

Divide 184,41475 by 32,5

```
32,5)184,41475(5,6743
     1625
     ----
      2191
      1950
      ----
       2414
       2275
       ----
        1397
        1300
        ----
         975
         975
         ---
           0
```

In this Example, there is five Decimals in the Dividend, and but one in the Divisor, then there must be 4 in the Quotient.

If you have any decimal Fraction to divide by a Unit, with any Number of Cyphers annex'd, it is but removing the Comma or Mark to distinguish the Decimal, as many Places to the left Hand, as you have Cyphers annexed to the Unit.

EXAMPLES.

Divide 14,4 by 10, the Quotient is 1,44.
Divide 1,44 by 10, the Quotient is ,144.
Divide 1,44 by 100, the Quotient is ,0144.
Divide 1,44 by 1000, the Quotient is ,00144.

N. B.

N. B. If the Dividend be greater than the Divisor, the Quotient will be either an Integer or a mixt Number; but if the Dividend be less than the Divisor, the Quotient will be a Decimal.

Multiplication and Division in Decimals (as in Integers) interchangeably prove each other.

To prove Multiplication, divide the Product by the Multiplier, gives in the Quotient the Multiplicand; or dividing the Product by the Multiplicand, gives in the Quotient the Multiplier.

To prove Division, multiply the Quotient by the Divisor, gives the Dividend.

What Divisor is that, by which dividing 746 shall give a Quotient equal to the Product of the same Number multiplied by 4? Facit ,25

Found by dividing an Unit with Cyphers by the Multiplicator thus.

```
4)1,00(,25
  8
  --
  20
  20
  --
   0
```

The

The Proof.

```
  749            ,25)749,00(2996
    4                50
 ----               ---
 2996               249
                    225
                    ---
                     240
                     225
                     ---
                      150
                      150
                      ---
                        0
```

In this the Product and Quotient are the same, and so of any other.

What Multiplicator is that by which multiplying 749, shall give a Product equal to the Quotient of the same Number divided by ,25? Facit 4.

Found by dividing Unity with Cyphers annexed by the Divisor, thus:

```
,25)1,00(4
    1 00
    ----
       0
```

If the Dividend be 6125, and the Quotient 49000, what is the Divisor? Facit ,125 found by dividing the Dividend with Cyphers, added by the Quotient.

If what has been ſaid in Decimal Fractions be underſtood, any Queſtion may be anſwered in any of the Rules of Three in Decimals, by obſerving the Rules given in Reduction, &c.

Single Fellowſhip

IS when two or more Men trade together with one common Stock, and when they have gained or loſt, it ſhews how much is each Man's particular Share of the Gain or Loſs, without any regard to Time. The Rule is this.

Rule: As the whole Stock is to the whole Gain or Loſs, ſo is each Man's particular Stock to his particular Gain or Loſs.

EXAMPLE.

Four Merchants, *A*, *B*, *C*, *D*, trading together, made a common Stock of Money, to the Value of 240 l. whereof *A.* put in 30 l. *B.* 50 l. *C.* 60 l. and *D.* 100 l. This Stock they occupied till they gained 2760 l. what did each Man gain.

All Queſtions in this Rule are anſwered by ſo many Statings in the Rule of Three, as there are Partners. See the Work of this Queſtion.

If 240 : 2760 :: 30 : 345 = A's Gain.
If 240 : 2760 :: 50 : 575 = B's Gain.
If 240 : 2760 :: 60 : 690 = C's Gain.
If 240 : 2760 :: 100 : 1150 = D's Gain.

The

The Anſwer is obtained by $\times$ the ſecond and third Term together, and dividing that Product by the firſt. And the Proof is adding up the ſeveral Gains into one Sum, which gives the total Gain = 2760.

EXAMPLE 2d.

If a Bankrupt's Goods be valued at 500 l. and he be 1000 l. in Debt to 4 Men, *viz.* *A*, *B*, *C*, *D*, of which 160 l. is due to *A*, 200 l. to *B*, 300 l. to *C*, 340 l. to *D*, now, what ought to be each Man's Share?

If 1000 : 500 :: 160 : 80 = A's Share.
If 1000 : 500 :: 200 : 100 = B's Share.
If 1000 : 500 :: 300 : 150 = C's Share.
If 1000 : 500 :: 340 : 170 = D's Share.

And ſo of any other Queſtion in this Rule.

Fellowſhip with Time

IS wrought with this Rule. Multiply each Man's particular Stock by his Time; then ſay, as the Sum of theſe Products is to whole Gain or Loſs, ſo is each Man's particular Product to his particular Gain or Loſs.

EXAMPLE.

Four Merchants *A*, *B*, *C*, *D*, made a common Stock, which at the Year's end was increaſed to 35,145 l. of this Stock *A* put in 669 l. which he took out at 10 Month's end; *B* laid in 810 l. for 8 Months; *C* laid in 900 l. for 7 Months; and *D* put in 1040 l. for 12 Months. Now what is each Man's Share of the Stock?

669 × 10 = 6690 A's Stock, into his Time.
810 × 8 = 6480 B's Stock, into his Time.
900 × 7 = 6300 C's Stock, into his Time.
1940 × 12 = 12480 D's Stock, into his Time.

31950

If 31950 : 35145 :: 6690 : 7359 = A's.
If 31950 : 35145 :: 6480 : 7128 = B's.
If 31950 : 35145 :: 6300 : 6930 = C's.
If 31950 : 35145 :: 12480 : 13728 = D's.

Proof 35145

For if you add each Man's Share of the Stock into one Sum, it will produce the whole Stock, if your Work be true.

EXAMPLE.

There is in a Cathedral Church, 20 Canons, and 30 Vicars, and they ſpend in a Year 2600 l. but every Canon muſt have five times as much as a Vicar, how much is each Man's yearly Income?

In anſwering ſuch Queſtions as this, you muſt multiply the Number of Perſons by their difference of Portions, and add the Products into one Sum, as you did by Time.

So that 20 × 5 = 100, to which add 30 = 130, then
If 130 : 2600 :: 100 : 2000 l. for the 20 Canons.
If 130 : 2600 :: 30 : 600 l. for the 30 Vicars.

And if you divide 2000 by 20, it gives 100 l. for each Canon; and 600 divided by 30, gives 20 l. for each Vicar.

The

The Rule of Alligation

TEacheth, how to mix or unite many Simples or Particulars, into one Sum or Maſs, according to any Price or Proportion required. I ſhall divide it into four Varieties, that when a Queſtion is propounded, you may ſee what Variety it falls under.

EXAMPLE.

If a Corn Factor mix 5 Buſhels of Wheat at 6 Shillings the Buſhel, with 12 Buſhels of Rye at 4 Shilling the Buſhel, with 8 Buſhels of Beans at 5 Shillings the Buſhel, and with 18 Buſhels of Barley at 2 Shillings and 6 Pence the Buſhel; the Price of one Buſhel of this Mixture is demanded?

5	Buſhels at 6s. the Buſhel, is	1	10	0
12	Buſhels at 4s. the Buſhel, is	2	08	0
8	Buſhels at 5s. the Buſhel, is	2	00	0
18	Buſhels at 2s. 6d. the Buſh, is	2	05	0
43	Buſhels. The total Value is	8	03	0

B. B.

Then ſay, if 43 : 8l. 3s. :: 1; or if you bring 3 Shillings into the Decimal of a Pound, it will ſtand thus:

B : L. B. L.

If 43 : 8,15 :: 1 : ,1895 = to 3s. 9d. $\frac{1}{4}$.

VARIETY 2d.

In this the Price of the Simples is expreſſed, but no Quantity given, and it is demanded how much

of each Simple we muſt take to ſell one Quantity or Meaſure at a Price propounded; the whole of this Variety is in linking the Extreams truly together, and taking the Differences betwixt them and the Mean; and theſe Differences are the true Quantities ſought. But inſtead of linking them, if you make the ſame Mark to any two you would join together, it will be full as plain and more quickly done.

Examples will make it eaſy.

Suppoſe a Man hath Spices, ſome at 9d. the Pound, ſome at 12d. ſome at 24d. and ſome at 30d. how much of each Sort muſt he take that he ſell a Pound for 20d.? Set down the Prices and Mean in the following Manner.

Mean 20	9x	10
	12✓	4
	24✓	8
	30x	11

Here you ſee that 9 and 30 are markt alike, and ſo is 12 and 24. And the Difference betwixt 9 and 20 is 11, which place to its like Mark 30, and the Difference betwixt 20 and 30 is 10, which place to 9, and the Difference betwixt 12 and 20 is 8, which place to 24; and the Difference betwixt 24 and 20 is 4, which place to 12; and the Differences thus found is the Anſwer to the Queſtion. For as often as he takes 10 Pound, of 9d. a Pound, he muſt take 4 of 12d. the Pound, and 8 of 24d. the Pound, and 11 of 30d. the Pound, as you may prove by the laſt Queſtion. You may have many Anſwers to ſuch Queſtions as this, if you mark one leſs than the mean Price with one greater. As in the Queſtion before

20	9X	4
	12✓	10
	24X	11
	30✓	8

now, 9 and 24 is markt alike, ſo is the Prices 12 and 30. And the Difference betwixt 9 and 20 is 11, which place to 24; and the Difference betwixt

twixt 24 and 20 is 4, which place to 9; and the Difference betwixt 12 and 20 is 8, which place to 30; and the Difference betwixt 30 and 20 is 10, which place to 12. And thoſe Differences are the true Anſwer to the Queſtion as before. And you may have other Anſwers if you vary the marking of the Prices by the above Direction.

N. B. If you add the Differences of thoſe two Anſwers together, the ſeveral Sums will be the Anſwer; or if you multiply the Differences by one common Multiplier, the ſeveral Products will be the Anſwer, as you may try at your Pleaſure.

A Merchant hath Wines; Canary at 24 d. the Quart, Sherry at 16 d. the Quart, and Malaga at 12 d. the Quart; how much of each Sort muſt he take to ſell a Quart for 18 d.?

	24✓	2.6	8
18	16✓		6
	12✓		6

This Queſtion you ſee admits but of one Way of marking, becauſe there is but one Quantity larger than the mean Price, which is markt to both the other Quantities, which are under the mean Price. So that he muſt have of the Canary 8 Quarts, and 6 Quarts of each of the other, or in that Proportion.

VARIETY 3d.

In this we have the the Price of all the Simples, and the Quantity of one given, to find the Quantity of all the reſt, ſo as one Meaſure or Quantity may bear a mean Rate or Price propounded. Which to do, obſerve the following Rule.

As the Difference ſtanding againſt the given Quantity, is to the reſt of any Differences beſides :: So is the

the given Quantity, to the Quantities ſought. Each to his reſpective Difference.

EXAMPLE.

A Tobacconiſt hath 30℔. of Tobacco, at 24 d. the Pound, which he would mix with ſome of 12 d. ſome at 9 d. and ſome at 7 d. and he would know how much of the ſaid leſs Prices he muſt mix with the 30 ℔. of the beſt, that he may ſell a Pound for 16d.? Set them down as in the laſt Example.

16	24✓	4. 7. 9.	20
	12✓	8	8
	9✓	8	8
	7✓	8	8

N. B. That 24 is markt to all the other, ſo you muſt take the Differences betwixt every Price and the Mean, and place it to 24, and the Sum of theſe is 20. Then you muſt take the Difference betwixt 24 and the mean Price which is 8, and Place it againſt every Price, becauſe 24 is markt to every Price; then ſay

If 20 : 8 :: 30 : 12

So that he muſt have 12 ℔. of each Quantity to mix with the 30 ℔. of the beſt, that a Pound may be ſold for 16d.

EXAMPLE 2d.

A Goldſmith hath 20 Ounces of Gold, at 20 Carracts fine, and would mix it with ſome at 22 Carracts fine, and ſome at 24 Carracts fine; how much

of

22 and 24 Carracts fine, and how much Alloy must he mix with the 20 Ounces of 20 Carracts fine, to have an Ounce, and consequently the whole Mass at 18 Carracts fine.

Note, That Alloy is a Sort of course Silver, or Copper, or some base Metal, with which Goldsmiths mix Gold or Silver, to abate the Fineness thereof.

An Ounce of Gold is divided into 24 Parts called Carracts, and an Ounce of Silver into 20 Parts, called Penny Weights; therefore to distinguish Fineness of Metals, such Gold as will abide the Fire without Loss, is accounted 24 Carracts fine; if it lose 2 Carracts in Trial, it is then called 22 Carracts Fine, &c.

Silver is valued in Ounces, and a Pound of Silver which loseth nothing in Trial, is called 12 Ounces fine; but if it loseth 2 Penny Weight, it is then said to be 11 Ounces, 18 Penny Weight fine. Set down the Values in Order, as usual, with the mean Value, and in the Place of the Alloy, (because it is not accounted of any Value,) place a Cypher, then take the Differences, which by marking you may see, will be the same, except only in the Place of Alloy.

18	20✓	18	18
	22✓	18	18
	24✓	18	18
	0✓	2.4.6	12

Then say If 18 : 18 : : 20 : 20

If 18 : 18 : : 20 : 20

If 18 : 12 : : 20 : 13$\frac{1}{3}$

So you see, that with the 20 Ounces of 20 Carracts fine, you must mix 20 of 22 Carracts fine, and 20 of 24 Carracts fine, and 13 Ounces and $\frac{1}{3}$ of Alloy, that so an Ounce will bear 18 Carracts fine. You may observe, that every Quantity

Quantity is markt with the ſame Mark as the Alloy; ſo that the Difference betwixt 20 and 18 is 2, betwixt 22 and 18 is 4, and betwixt 24 and 18 is 6, whoſe Sum is 12, then the Difference betwixt the Alloy and 18 is 18, which place againſt every Quantity, as it has the ſame Mark.

VARIETY 4th.

In this the Prices of every Simple is expreſſed, and the mean Rate or Price; and it is required to find how much of each Sort muſt be taken to make up a certain Quantity propounded, agreeable to the mean Rate given? Which to do, obſerve the Rule following.

As the total Sum of the Differences is to the total Quantity given, ſo is any particular Difference to its particular Quantity ſought.

EXAMPLE.

A Merchant would mix 4 Sorts of Wine, of ſeveral Prices, one at 6 Shillings *per* Gallon, another at 8 Shillings, the third at 11 Shillings, and the fourth at 15 Shillings *per* Gallon; of theſe Wines he would have a Mixture made to contain 50 Gallons, and the Price of one Gallon to be 9 Shillings. How much muſt he take of every Sort of Wine?

	6✓	6
9	8×	2
	11×	1
	15✓	3
		12 = Sum of the Differences; then ſay,

If

If 12 : 50 :: 6 : 25=the Gal. at 6s *per* Gallon
If 12 : 50 :: 2 : $8\frac{1}{3}$=the Gal. at 8s *per* Gal.
If 12 : 50 :: 1 : $4\frac{1}{6}$=the Gal. at 11s *per* Gal.
If 12 : 50 :: 3 : $12\frac{1}{2}$=the Gal. at 15s *per* Gal.

50 Proof.

For the $\frac{1}{3}$ and $\frac{1}{6}$ of any Thing is = to $\frac{1}{2}$. You may have other Anſwers, if you vary the marking of the Quantities by the Rule given in the ſecond Variety. And the Proof of the Money is found by the firſt Variety.

Extraction of the Square Root.

THE Square Root of a Number, is a Number which being multiplied by itſelf, produceth the given Number; or, the Square Root of a Number may be defined to be a mean Proportional between the given Number and Unity. Here follows a Table of Squares as far as the firſt 9 Digits.

Root	1	2	3	4	5	6	7	8	9
Square	1	4	9	16	25	36	49	64	81

And when it is required to extract the Square Root of any given Number, we have nothing to do but to find that equal Number of which it was compoſed; ſo if the Root of 25 were required, it would be found 5 for $5 \times 5 = 25$, as you may ſee by the Table. Here 5 is the Root or firſt Power, and the Number

25 is called the Square, or second Power. Of Numbers to be extracted, are three Sorts. First Single, as in the Table, which are composed of the 9 Digits.

Secondly, Compound, that are composed of more Figures than one, as of 16, whose Square is 256, for 16×16=256, or 48×48=2304, or 100×100=10000. Here the Roots are 16.48 and 100, and the Squares 256.2304 and 10000. Third, Irrational are all such Squares, whose Roots cannot be discovered by Art exactly, neither in whole Numbers or Fractions; but something will still remain, there being no Proportion found betwixt an irrational Number and its Root, such Numbers are 5. 19. 21. 75. 156. 867. &c. The Extraction of the Square Root is not much unlike Division, only there our Divisor is fixed, here we are to find a new one for each Opperation. The Root of any single Square Number is found by Inspection, as may be seen by the foregoing Table. But if it be a compound Square Number, it must be prepared by pointing thus: make a Point under the Units Place, and omitting one, point every other Figure; and as many Points as your Number contains, so many Figures will your Root consist of.

Examples will make it easy; but if the Learner first take a Number, and square it, and extract the Root of the Product, it will facilitate the Manner used in the Extraction.

EXAMPLE 1ſt.

If 246×246=60516, then find the Root of 60516, which being pointed as directed, it will ſtand thus:

```
              60516(246=the Root
              4
              —
Root × 2 = 44)205
              176
              ———
Root × 2 = 486)2916
               2916
               ————
                  0
```

You know at the Beginning of the Work what the Root will be, then point it, beginning at the Units Place, and miſs every other Figure, and the Points will ſtand under the Numbers 6. 5. and 6. which ſhews you that there will be three Figures in the Root = to the Number of Points. Then take the neareſt Square by your Table, which is the ſame as your laſt Point, or under it in Value, as you will find 2, which place in the Root, and ſet its Square under the 6, and ſubtract it, and the Remainder is 2; then bring down your next Period 05, and put it to the Remainder, and it will be 205; then draw a Line as in Diviſion, and × your Root by 2, and it makes 4, which place as your Diviſor, leaving Room towards the right Hand, and place your next Figure, that will be 4, in your Root in that Place; then multiply as in Diviſion, and place the Product, which is 176 under 205, and the Remainder will be 29, to which bring down your next Period 16, and join it to the Remainder, and it will make 2916; then draw a Line as before, to place your Diviſor in,

and double your Root as before, and it will be 48, ſtill leaving Room towards the right Hand to place the next Figure in your Root there, which is 6, and the Diviſor is then 486, which being × by 6, the laſt Figure in the Root, the Product is 2916, and there is no Remainder; which ſhews that your Work is right, Obſerve this in all the following Examples.

EXAMPLE 2d.

176×176=30976, and its Root is required to be found by the Method uſed in extracting the Square Root; it being pointed, the Work will ſtand in this Manner:

```
30976)176 Root
1
—
27)209
   189
   ———
346)2076
    2076
    ————
       0
```

EXAMPLE 3d.

674×674=454276

What is the Square Root of 454276? See the Work.

```
     454276(674=Root.
     36
    ------
 127)942
     889
    --------
1344)5376
     5376
     ------
        0
```

EXAMPLE 4th. with Decimals

87,25×87,25=7612,5625

What is the Square Root of 7612,5625? See the Work.

```
        7612,5625)87,25=Root.
        64
        ----
   167)1212
       1169
       ------
  1742)4356
       3484
       ------
 17445)87225
       87225
       ------
           0
```

N. B.

N. B. If there had been an odd Number of Decimals, you must have put a Cypher towards the right Hand, to make them even, and then begun to point as directed; or if you begin at the right Hand of the Decimals, it would have been the same, and the Points shew you the Number of whole Numbers and Decimals that will be in the Root. For those Points under the Decimals, shew the Number of Decimals in the Root, and those under the whole Numbers, the whole Numbers.

EXAMPLE 5th.

17,75×17,75=315,0625

What is the Square Root of 315,0625?

```
315,0625(17,75=Root.
1
-----
27)215
   189
-----
347)2606
    2429
-----
3545)17725
     17725
-----
         0
```

If the Learner observe the Directions given, by this Time the Method of extracting the Square Root

will be familiar to him; and I shall now proceed to extract the Root of a Number, without knowing from what Number it grew.

What is the Root of 451584(672=Root?

```
        36
    127)915
        889
   1342)2684
        2684
           0
```

N. B. If you multiply the Root by itself, the Product will be the Number given to be extracted; and if there be a Remainder, you must add it to the Product, and the Sum will be the given Number, if your Work is right.

Extract the Square Root of 4624(68=Root.

```
        36
    128)1024
        1024
           0
```

What

What is the Root of 784762(885

```
         784762(885
         64
        ----
    168)1447
        1344
        ------
   1765)10362
         8825
        ------
         1537 Remainder.
```

Now $885 \times 885 = 783225 + 1537$ the Remainder $= 784762$; which proves the Work right.

N. B. If you add two Cyphers to the Remainder, you might have found the first Place of Decimals, by proceeding in every Respect as before; and then two more Cyphers being added to the next Remainder, and proceeding as before, you would have had the second Place in Decimals; and by adding two Cyphers to every Remainder, you might bring the Root to as many Places of Decimals as you please.

But if you would have your Root expressed as a Vulgar Fraction, do thus: After you have found the Integral Part of your Root, to its Quadruple, add Unity for the Denominator of the Fractional Part, and the Remainder doubled is the Numerator; so the Root, by this Method will be $885\frac{3074}{3541}$ nearly.

Extract

Extract the Root of 674637

```
674637(821,36289
64
----
162)346
    324
    ---
1641)2237
     1641
     ----
16423)59600
      49269
      -----
164266)1033100
        985596
        ------
1642722)4750400
        3285444
        -------
16427248)146495600
         131417984
         ---------
164272569)1507761600
          1478453121
          ----------
            29308479
```

In this Example, the Root is extracted to five Places of Decimals, which is near enough: for if Unity was divided into a Thousand Parts, it would not want three of them, and if it was carried two Decimals more, it would not want 2 Parts if Unity were divided into a Hundred Thousand Parts; as you may prove at your Pleasure.

Questions to Exercise the Square Root.

TO find a mean Proportional between any two Numbers.

R U L E.

The Square Root of the Product of the given Numbers, is the mean Proportional sought.

E X A M P L E. 1st.

Find a mean Proportional betwixt 49 and 64.
Then 49 × 64 = 3136.
And 3136(56 = The Root and mean sought.

```
 25          For as 49 : 56 :: 56 : 64.
 ---
106)636
    636
    ---
     0
```

To find the Side of a Square = in Area to any given Superficies whatsoever.

R U L E.

Find the Square Root of the Area, and that Root is the Side of the Square sought.

If the Content of a Circle be 160, what is the Side of a Square = in Area to the given Circle?

160

160(12,64911 = the Side of a Square, equal in Area to the given Circle.
1

22)60
44

246)16,00
14 76

2524)12 400
10 096

25289)23 0400
22 7601

252981)279900
252981

2529821)2691900
2529821

162079 Remainder.

N. B. After you have brought out the Integers 12, and would have the Root expressed as a Vulgar Fraction, by the Rule given before, you will find it to be 12 $\frac{32}{49}$, but the Decimals is the most exact. And 12,64911 is the Side of a Square, whose Area is 160 Square Perches, or one Acre.

Any Number of Soldiers being proposed, to order them into a Square Battalia; that is, there shall be as many in Rank as in File. *Rule.* Extract the Square Root of the given Number of Men, and that Root is the Answer.

Suppose

Suppose a General had an Army of 138384 Men, and he would form them into a Square Battalia; how many Men must be in Rank and File?

$$\begin{array}{l} 138384(372 = \text{the No. in Rank and File.} \\ 9 \\ \hline 67)483 \\ 469 \\ \hline 742)1484 \\ 1484 \\ \hline 0 \end{array}$$

Any Number of Men being proposed to order them into a double (triple, or Quadruple, &c.) Battalia, that is, having twice (or three times, or four times, &c.) as many in Rank as in File.

R U L E.

Extract the Square Root of $\frac{1}{2}$ (or $\frac{1}{3}$ or $\frac{1}{4}$, &c. the given Number of Men, and that is the Number of Men in File, and double (or triple, or Quadruple, &c.) that Number, is the Number in Rank.

Let 8192 Men be formed into an Oblong Battalia, so that the Number of Men in Rank may be double the Number of Men in File? And $8192 \div 2 = 4096$. Then 4096(64 = the Number of Men in File and $64 \times 2 = 128$ the No. in Rank.

$$\begin{array}{l} 4096(64 \\ 36 \\ \hline 124)496 \\ 496 \\ \hline 0 \end{array}$$

Any

Any Number of Men being proposed to be put in Battalia, and a certain Number assigned to be put in Rank, to find the Number in File; or a certain Number assigned to be put in File, to find the Number in Rank.

R U L E.

As the Number of Men assigned to be put in Rank, is to the whole Number of Men; so is Unity to the Number in File. And as the Number of Men assigned to be put in File is to the whole Number of Men; so is Unity to the Number in Rank.

Let 12132 Men be placed in Battalia, so that there shall be 18 in Rank; how many must be in File?

As 18 : 12132 :: 1 : 674 to the Number in File.

Any Number of Men being given to be put in Battalia, and their Number and Distance in Rank and File being given to find how much Ground they will occupy. As 1 is to the Distance in Rank; so is the Number in Rank less 1 to a 4th. And as 1 is to the Distance in File, so is the Number in File less 1 to a 4th. Then the Rectangle or Product of these 4ths, gives the Quantity of Ground sought, in the same Measure as the Distance was given.

E X A M P L E.

Let it be required to place 6068 Men in Battalia, so that there be 82 in Rank, and 74 in File, and they

they all to stand in Order, that is, three Feet distant in Rank, and as much in File; how much Ground will they occupy? For

As 1 : 3 :: 81 : 243 } And 243 x 219 = 53217. And,
As 1 : 3 :: 73 : 219 }

So many Square Feet of Ground they will occupy.

Though these two last Questions do not belong to the Square Root, I thought it would not be disagreeable to give them a Place here, as they relate to Military Affairs.

Any Number of Men being proposed, to place them in Battalia, so that the Number of Men in Rank, shall be to the Number of Men in File, according to any Proportion assigned; or the Number in File to the Number in Rank.

R U L E.

As the Ratio respecting the Rank, is to the Ratio respecting the File; so is the whole Number of Men to a 4th, whose Square Root is the Number of Men in File, &c.

Let 864 Men be placed in Battalia, so the Number of Men in Rank may be to the Number in File as 3 to 2; how many must there be in Rank, and how many in File?

Then as 3 : 2 :: 864 : 576, whose Root is 24 Number in File.

And as 2 : 3 :: 864 : 1296, whose Root is 36 Number in Rank.

The

The Area of a Circle being given, to find the Diameter.

R U L E.

AS 355 is to 452, or as 1 to 1,273239, so is the Area to the Square of the Diameter.

What Length of Cord will serve to tye to a Cow-Tail, the other End fixed to the Ground, to let her have Liberty of eating an Acre of Grass and no more, supposing the Cow and Tail to be one Perch, or five Yards and a Half?

Say, as 355 : 452 :: 160 being the Area of a Circle, whose Content is an Acre, to 203,7183. whose Square Root is 14,2732, the Diameter sought, whose Radius is 7,1366, from which subtract one Peach for the Cow and Tail; rest 6,1366 for the Length of the Cord sought.

The Area of a Circle given to find the Periphery.

R U L E.

Say as 113 : 1420 :: or, as 1 to 12,56637, so is the Area to the Square of the Periphery. So if the Area of a Circle be 160, the Periphery will be found 44.84.

A certain Number of Men spent at a Reckoning the Sum of 14l. 8s. and every Man paid as many Sixpences as there were Men in Company. How many Men were there?

Rule.

R U L E.

Bring the Money into Sixpences, and extract the Root of that Product, which Root will be the Number of Men. See the Work.

l.	:	s.
14	:	8
20		

288
2

576 (24=the Number of Men.
4

44)176
176

0

In the 47th Proposition of the first Book of Euclid, it is proved that the Square of the Hypothenuse, or longest Side of a right-angled Triangle, is equal to the Sum of the Squares of the Base or Perpendicular; or the other two Sides.

Let the Base of a right angled Triangle be 36 Feet, and the Perpendicular Height be 48 Feet, what Length will the Hypothenuse, or longest Side of the Triangle be?

$36 \times 36 = 1296$. And $48 \times 48 = 2304$.
Then $1296 + 2304 = 3600$, whose Square Root is 60, the Length of the longest Side in Feet sought.

The

The Hypothenuſe, or longeſt Side of a right angled Triangle be 60 Feet, and the Baſe 48 Feet, what is the Perpendicular?

$60 \times 60 = 3600$. And $48 \times 48 = 2304$. Then $3600 - 2304 = 1296$, whoſe Root is 36 the Length of the Side ſought.

Let the Hypothenuſe of a right angled Triangle be 60 Feet, and the perpendicular Height 36 Feet, what is the Baſe Line?

$60 \times 60 = 3600$. And $36 \times 36 = 1296$. Then $3600 - 1296 = 2304$, whoſe Root is 48 Feet, the length of the Baſe Line.

Thoſe three Queſtions mutually prove one another. And this much ſhall ſuffice for the Square Root, and I ſhall proceed to the Cube.

Extraction of the Cube Root.

THE Cube Root of a Number, is a Number, which, being multiplied by itſelf twice, produceth the given Number. Thus the Cube Root, of 125 is 5: For $5 \times 5 \times 5 = 125$. And the Cube Root of 512 will be found 8. And ſo of other Numbers, as in the following Table.

The TABLE.

Root.	Cube.
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729

The Root of any single Cube Number is found by Inspection, as in the Table.

But if it be a compound Cube Number, it must be pointed thus; make a Point under the Units Place, and omitting two, point every third Figure, and as many Points as your Number contains, so many Figures will your Root consist of. Then proceed by the following Direction.

Take the nearest Cube Number to your first Period to the left Hand, by the Table, whose Root place in the Quotient, then subtract its Cube from the Period, and to the Remainder (if any) bring down three Figures, or your next Period, which call your Dividend, then call the Quotient Figure, with a Cypher joined to it, *r*; and square it, and × that Square by 3, and the Product you may call your Divisor. And then proceed as in Division, only with this Difference, call your next Quotient

Figure *e*, and × your Divisor by it, and place it under your Dividend, then × your *r* by 3, and that Product × by the Square of that Figure you call'd *e*; lastly cube the Figure you call'd *e*, and place those two last under the Dividend also, and add the three several Sums into one, and subtract this last from your Dividend, and to the Remainder bring down your next Period, and proceed as before. Examples will make it more intelligible than a multitude of Words.

Cube 68, that is 68×68×68=314432.

Then extract the Cube Root of 314432. Point it as directed, and it will stand thus 314432 (68

216

60=*r*, then 3*rr* that is

60×60×3=10800)98432=Divid.

86400=3*rre*

11520=3*ree*

512=*eee*

98432

0

N. B. That 8 the second Figure in your Quotient is called *e*; and if your Root consist of more Figures than two, then the *r* changes its Name every Period you bring down, as you may may see in the following Example.

EXAMPLE 2d.

$672 \times 672 \times 672 = 303464448$

What is the Cube Root of 303464448? See the Work.

303464448(672
216

$3rr = 10800$)87464=the Dividend.

75600=$3rre$
8820=$3ree$
843=eee

84763=The Subducend.

$3rr = 1346700$)2701448=The Dividend.

2693400=$3rre$
8040=$3ree$
8=eee

2701448=The Subducend.

0

When you bring down your ſecond and laſt Period, the r becomes 670, and your e 2, and ſo proceed to change your r every Period you bring down.

Find

Find the Cube Root of 307273580992? See the Work.

$$
\begin{array}{rl}
 & 307273580992(6748=\text{Root.} \\
 & 216 \\
\hline
3rr=10800) & 91273=\text{Dividend.} \\
\hline
 & 75600=3rre,\ \text{here}\ e=7: \\
 & 8820=3ree \\
 & 343=eee \\
\hline
 & 84763=\text{Subducend.} \\
\hline
3rr=1346700) & 6510580=\text{Dividend the 2d.} \\
\hline
 & 5386800=3rre,\ \text{here}\ e=4. \\
 & 32160=3ree \\
 & 64=eee \\
\hline
 & 5419024=\text{Subducend.} \\
\hline
3rr=136282800) & 1091556992=\text{Dividend 3d.} \\
\hline
 & 1090262400=3rre\ \text{here}\ e=8 \\
 & 1294080=3ree \\
 & 512=eee \\
\hline
 & 1091556992=\text{Subducend.} \\
\hline
 & \text{Rest}-00
\end{array}
$$

In this Example your first r was $=60$, your first $e=7$, your second $r=670$, your second $e=4$, and your

your last r=6740, and your last e 8; and in this Manner you must go on till your Work is finished, and if there had been a Remainder, you must add three Cyphers to it for a Decimal, &c.

Extract the Cube Root of 282; or find the Side of a cubical Vessel, which shall contain a Gallon of Ale, being 282 solid Inches?

282(6,557=Root.
216

$3rr$=10800)66000 Dividend.

54000=$3rre$
4500=$3ree$
125=eee

58625=Subducend.

$3rr$=1267500)7375000=Dividend the 2d.

6337500=$3rre$
48750=$3ree$
125=eee

6386375=Subducend.

$3rr$=128707500)988625000=Dividend 3d.

900952500=$3rre$
962850=$3ree$
343=eee

901915693 Subducend.

Remainder 86709307

This

This is carried to three Places of Decimals, and the Work is all the same as in whole Numbers, only making the Comma, to distinguish the Decimals, when you bring down the first three Cyphers to the Remainder.

If you have the Cube Root of any Fraction to find, reduce it into Decimals of 3, 6, 9, or 12 Places, as you desire your Root to be less or more exact.

What is the Cube Root of $\frac{1}{2}$, the Work to three Places of Decimals, will stand in the following Manner:

,500000000(,793 Root.
343

3*rr*=14700)157000 Dividend.

132300=3*rre*
17010=3*ree*
729=*eee*

150039 Subducend.

3*rr*=1872300)6961000=Dividend.

5616900=3*rre*
21330=3*ree*
27=*eee*

5638257 Subducend.

Remainder 1322743

To

To prove if your Work be true, Cube the Root, and to that Cube add the Remainder, and that Sum will be the same as the Number given to be extracted.

Proof of the last Example.

	,628849
	,793
,793	1886547
,793	5659641
2379	4401943
7137	
5551	498677257 Cube.
	1322743 Rest.
,628849 Square.	
	,500000000 Proof.

But if the mixt Number or Fraction be commensurable to its Root, then extract the Cube Root of the Numerator for the Numerator of the Root, and the Cube Root of the Denominator for the Denominator of the Root; so the Cube Root of $\frac{8}{27}$ will be $\frac{2}{3}$, for the Cube Root of 8 is 2, and of 27 is 3; which is $\frac{2}{3}$ and so of any other.

Questions

Questions to exercise the Cube Root.

IF a Bullet, whose Diameter is 4 Inches, weigh 9 Pound, what will a Bullet of the same Metal weigh, whose Diameter is 8 Inches?

R U L E.

Like Solids are in triple Proportion to their homologous Sides, Diameters, Lines, &c.; it will be as the Cube of the Diameter is to its given Weight, so is the Cube of the other Diameter to the Weight sought.

C.D. : lb. :: C.D. : lb.

If 64 : 9 :: 512 : 72. For $512 \times 9 = 4608 \div 64 = 72$ lb.

If the Diameter of a Globe be 1 Inch, and the Solidity thereof be ,5238 of an Inch; what is the Solidity of another Globe, whose Diameter is 10 Inches?

C.D. : S. :: C.D. : S.

If 1 : ,5238 :: 1000 : 523,8 Inches. For $,5238 \times 1000 = 523,8000 \div 1 = 523,8$ the Solidity.

If a Globe of Silver of 1 Inch Diameter, be worth 14,4 Shillings, what will another Globe of Silver be worth, whose Diameter is 4 Inches?

If 1 : 14,4 :: 64 : 921,6. For $14,4 \times 64 = 921,6 \div 1 = 921,6$ Shillings, that is 46l. 1s. 7d$\frac{1}{5}$.

If a Saker of 3,75 Inches Diameter at the Bore, require 4 ℔. of Powder for its Charge; what will a Demi-Canon of 6,5 Inches in the Bore, require for its Charge?

C. D. ℔. C. D.
If 52,734375 : 4 :: 274,625 : 20,83.
For 274,625 × 4 = 1098,500 ÷ 52,734375 = 20,83 ℔. the Anſwer.

If a Braſs Saker, whoſe Diameter is 11,5 Inches, do weigh 1000 ℔. what will another Piece of Ordinance (of ths ſame Metal and Shape) weigh, whoſe Diameter is 8,75 Inches?

C. D. : ℔. :: C. D. ℔.
If 1520,875 : 1000 :: 669,921875 : 440,484

If a Ship, whoſe Burthen is 300 Tun, be 75 Feet in the Keel, what ſhall be the Burthen of another Ship (of the the ſame Mold) whoſe Keel is 100 Feet?

C. K. : Tun :: C. K. Tun.
If 421875 : 300 :: 1000000 : 711,111.

If a Bullet, whoſe Diameter is 4 Inches, weigh 9 ℔. what is the Diameter of another Bullet (of the ſame Metal) whoſe Weight is 72 ℔.? Anſwer 8 Inches. For,

℔. : C.D. ℔. C.D.
If 9 : 64 :: 72 : 512; whoſe Cube Root is 8 Inches the Diameter.

If

If the Diameter of a Globe be 1 Inch, and the Solidity ,5238 of an Inch, what is the Diameter of another Globe, whose Solidity is 523,8 Inches? Anſwer, 10 Inches. For,

	Solid	C. D.	: :	Solid	:	C.D.
If	,5238 :	1	: :	523,8	:	1000; whoſe Cube

Root is 10 Inches.

If a Globe of Silver of 1 Inch Diameter be worth 14,4 Shillings, what will be the Diameter of another Globe of Silver that is worth 921,6 Shillings. Anſwer, 4 Inches. For,

	S.	C.D.		S.		C.D.
If	14,4 :	1	: :	921,6	:	64; whoſe Cube

Root is 4.

If you would have the Side of a Cube = in Solidity to any given Solid, as Globe, Cylinder, Priſm, Cone, &c the Cube Root of the Solid Content will be the Side. But if you would have the Side double, treble in Quantity, × the Solidity by 2, 3, &c, if leſs, divide by 2, 4, &c. and the Cube Root of the Product or Quotient is the Anſwer.

Between 2 Numbers given, to find 2 mean Proportionals. *Rule*, × the leſs Extream by the Cube Root of the Quotient of the greater ÷ by the leſs; the Product is the leſſer of the 2 mean Proportionals, which × by the ſaid Cube Root, gives the greater Mean ſought.

Find 2 mean Proportionals betwixt 8 and 512. Then $512 \div 8 = 64$, whose Cube Root is 4. And $8 \times 4 = 32$ the less Mean. And $32 \times 4 = 128$ the greater Mean.

Proof $8 : 32 :: 128 : 512$.

Many more Uses might be named, which the Learner will find by the Examples given, if they be underſtood.

The End of the First Part.

P A R T II.

THE

ALGEBRAIST'S COMPANION.

Explanation of the Signs made Use of in this Algebra.

$+$	Signifies Addition, and is termed Plus, or more.
$-$	Signifies Subtraction, and is called Minus, or less.
$\times$	Signifies Multiplication.
$\div$	Signifies Division, or $\frac{24}{4}$, or $\frac{x \times y}{n}$
@	Signifies Involution.
yv	Signifies Evolution, or the Root to be extracted.
$::$	Signifies Proportion.
$\div\!\!:$	Signifies continued Proportion.
$=$	Signifies Equality, or equal to
$\surd$	Signifies the Root to be extracted.
$C\,\square$	Signifies the Square must be compleated.

AXIOMS.

AXIOMS.

First, If to equal Things are added equal Things, their Sums are equal.

Second, If from equal Things, equal Things are taken away, their Remainders will be equal.

Third, If equal Things be multiplied by equal Things, their Products will be equal.

Fourth, If equal Things be divided by equal Things, their Quotients will be equal.

Fifth, Things equal to one and the same Thing, are equal to one another.

ALGEBRA

IS a specious Arithmetic, or an Arithmetic in Letters; it consists of Addition, Subtraction, Multiplication, Division, Involution, and Evolution, &c. or it is the Art of abstract Reasoning upon Quantity, by general and indefinite Representations, in order to resolve Problems, invent Theorems, and to demonstrate both.

ADDITION.

Rule 1st. If Quantities are alike, and have the same Sign before them, to be added together, put down the Sum of the Coefficients with the common Sign before them, and the Common Letter after them.

EXAMPLES.

EXAMPLES.

	Steps.				
	1	x	$-5a$	$7x$	$3x+2y$
	2	$2x$	$-4a$	$3x$	$2x+5y$
$1+2$	3	$3x$	$-9a$	$10x$	$5x+7y$

N. B. $1+2$ in the Margin, opposite the third Step, shew you, that the first and second Steps are added together, and the Sum is placed against the third Step.

Rule 2d. If the Quantities to be added have different Signs before them, then put down the Difference of the two Coefficients, with the common Letter after it, and the Sign of the greater Quantity before it.

EXAMPLES.

	1	$4x$	$3b$	$-12a$	$-12xy-7bc+16$
	2	$-3x$	$3b$	$7a$	$16xy+7bc-30$
$1+2$	3	x	*	$-5a$	$4xy$ * -14

To explain why an Affirmative Quantity added to a Negative, is made so much less, as the Negative is in Value, let us imagine a Person to have $40x$ Pounds; this Quantity represents the Value of his ready Money, Lands, Plate, Debts, &c. in his Possession, or owing to him; this $40x$ would be the real Sum the Person is worth, if there were no Debts due from him to others. But let us suppose he owes

$17x$,

$17x$, which, tho' it be something real, is yet of a direct contrary Nature to what is due to him, and must be expressed with a direct contrary Sign, and in the Nature of Debts, must destroy so much of the Person's Estate, as the Debt is in Value. Hence, if the Debts equal the Estate, they destroy each other, and the Person is worth nothing. If the Debts exceed the Estate, the Person is then worse than nothing; but if the Estate exceeds the Debts, it is an Affirmative, and the Person's real Worth is the Difference.

Rule 3d. If the Quantities to be added be of different Kinds, and such as will not incorporate, put them down in any Order, one after another, with their proper Signs before them; and this is all the Addition they are capable of. The Addition of compound Algebraick Quantities, is performed by collecting the several Members of every particular Species into as many Sums as there are Species; and then putting down the Sums in any Order, with their proper Sign before them.

EXAMPLES.

	1	x	a	$7m+12x+4dc$
	2	y	$-b$	$4a-20$
$1+2$	3	$x+y$	$a-b$	$7m+12x+4dc+4a-20$

	1	$3xx+4abc-bb+20$
	2	$2bb-3xx-2abc-15$
	3	$2dd-2bb+abc+7$
$1+2+3$	4	$3abc-bb+12+2dd$

Subtraction

Subtraction of Algebra

IS performed by changing the Sign of every particular Member of the Quantities to be subtracted, *i. e.* making every negative Member affirmative, and every affirmative Member negative, and then adding it to the other, (or supposing them changed) for since it has been already observed, the subtracting any one Quantity from another, is the same in Effect as adding its contrary; and since changing the Sign of every Member to be subtracted, renders that Quantity just the contrary to what it was before, it is evident, that after this Change it may be added, and that the Sum of this Addition will be the true Difference.

EXAMPLES.

	1	$2x$	$-2x$	$12xy$	$5a+12b$
	2	x	$-x$	$3xy$	$2a+8b$
1—2	4	x	$-x$	$9xy$	$3a+4b$

	1	$-10a+4x+7m-8n$
	2	$+10a+10x+8m-8n$
1—2	3	$-20a-6x\quad-m\quad *$

Multiplication of Algebra.

R U L E.

IF the Multiplicator and Multiplicand have Signs both alike, that is both affirmative, or both negative, the Product will then be affirmative: But if one be affirmative and the other negative, then the Product will be negative. Multiplication of simple Algebraick Quantities, is performed, first, by multiplying all the numeral Coefficients together, and then putting down after the Product, the Letters in both Factors, the Sign of the Product being prefixed to it. The Multiplication of compound Algebraick Quantities, is performed by multiplying every particular Member of the Multiplicator into all those of the Multiplicand, and then reducing the whole into the least Compass possible.

E X A M P L E S.

	1	x	$4x$	$+4nr$	$-7m$	$-12xr$
	2	y	$7ym$	$-7y$	$+3n$	$-3m$
1×2	3	xy	$28xym$	$-28nry$	$-21mn$	$+36xrm$

	1	$x+m$
	2	$x+m$
$1\times x$	3	$xx+mx$
$1\times m$	4	$+mx+mm$
$3+4$	5	$xx+2mx+mm$

$$\begin{array}{r|c|l}
 & 1 & 4x-5a-2b \\
 & 2 & 3x-2a+5b \\
\hline
1\times 3x & 3 & 12xx-15ax-6xb \\
1\times 2a & 4 & \quad -8ax+10aa+4ab \\
1\times 5b & 5 & \qquad\qquad 20xb-25ab-10bb \\
\hline
3+4+5 & 6 & 12xx-23ax+10aa+14xb-21ab+10bb
\end{array}$$

Division of Algebra

IS only the Reverſe of Multiplication, and all the Operations in this Rule may eaſily be perform-ed, by conſidering that the Quotient multiplied by the Diviſor gives the Dividend.

Rule 1ſt. If the Quantities have like Signs, but no Coefficients, expunge the Quantities in the Dividend that are in the Diviſor, and put the Sign $+$ before the Quotient.

EXAMPLES.

$$\begin{array}{r|c|cccc}
 & 1 & xy & -xy & am+bm & -ax-bx \\
 & 2 & x & -x & m & x \\
 & & \overline{} & \overline{} & \overline{} & \overline{} \\
1\div 2 & 3 & y & +y & a+b & -a-b
\end{array}$$

Rule 2d. If Quantities have Coefficients, divide them as in common Arithmetick, and expunge the Quantities as before.

EXAMPLES.

	1	$18ax$	$49xy$	$12am-24xm$
	2	$6x$	$-7x$	$6m$
$1 \div 2$	3	$3a$	$-7y$	$2a-4x$

Rule 3d. If the Divisor and Dividend be the same, the Quotient is one.

EXAMPLES.

	1	xy	$9\,mr$	$7\,an+4\,nm$
	2	xy	$-9\,mr$	$7\,an+4\,nm$
$1 \div 2$	3	1	-1	1

Rule 4th. If the Quantities in the Divisor are not like them in the Dividend, put them down as you do a vulgar Fraction.

EXAMPLES.

Divide xmy by nr; place it thus $\frac{xmy}{nr}$

Divide $5x+4y$ by $5n+r$; it will stand thus $\frac{5x+4y}{5n+r}$

If the Learner accustom himself to ask these three Questions, it will make Division easy. First, what Sign $\times$ into the Divisor, will give the Sign prefixed to

to the Dividend; secondly, what numeral Coefficient of the Quotient, × into the numeral Coefficient of the Divisor will make that of the Dividend; and thirdly, what Letters × into those of the Divisor, will make those of the Dividend? The Signs, Coefficients, and Letters, which arise upon asking these Questions, will be the true Quotient sought, as well in compound as simple Algebraick Quantities.

EXAMPLES.

$$-3a)12ab(-4b \qquad -6xy)-24yxxx(+4xx$$

$$\begin{array}{ll} 12ab & -24yxxx \\ \hline * & * \end{array}$$

$$\begin{array}{l}
2y+3x)16yyyy-72xxyy+81xxxx(8yyy-12xyy-18xxy+27x^3 \\
\quad 16yyyy+24xyyy \\
\hline
\qquad -24xyyy-72xxyy \\
\qquad -24xyyy-36xxyy \\
\hline
\qquad\quad * \quad -36xxyy+81xxxx \\
\qquad\qquad\quad -36xxyy-54xxxy \\
\hline
\qquad\qquad\quad * \quad +54xxxy+81xxxx \\
\qquad\qquad\qquad\quad 54xxxy+81xxxx \\
\hline
\qquad\qquad\qquad\quad * \qquad *
\end{array}$$

Involution

INVOLUTION

IS the Multiplication of like Quantities, or the raising of any given Quantity to any desired Power.

EXAMPLES.

	1	x	xx	xxx	$xxxx$	$xxxxx$
	2	x	x	x	x	x
1 ⊙ 2	3	xx	xxx	$xxxx$	$xxxxx$	$xxxxxx$
or thus		x^2	x^3	x^4	x^5	x^6

This $a \times a = aa$, or a^2, is a Square. And $aa \times a = aaa$, or a^3, is a Cube. $xxx \times x = xxxx$, or x^4, is Biquadrate of x; or the fourth Power of x, &c.

	1	$x+y$ This is called a Binomial Root.
	2	$x+y$
1 × x	3	$xx+xy$
1 × y	4	$xy+yy$
1 ⊙ 2	5	$xx+2xy+yy$ The Square of $x+y$.
		$x+y$
5 × 5	6	$xxx+2xxy+xyy$
5 × y	7	$xxy+2xyy+yyy$
1 ⊙ 3	8	$xxx+3xxy+3xyy+yyy$. The Cube of $x+y$.

1+x.

	1	$x-y$. This is called a residual Root.
	2	$x-y$
$1\times x$	3	$xx-xy$
$1\times y$	4	$-xy+yy$
1 ● 2	5	$xx-2xy+yy$. The Square of $x-y$.
		$x-y$
$5\times x$	6	$xxx-2xxy+xyy$
$5\times y$	7	$-xxy+2xyy-yyy$
1 ● 3	8	$xxx-3xxy+3xyy-yyy$. The Cube of $x-y$.

One may involve in this Manner to any Power whatever, by the Multiplication of any Power by its Root.

Let $x+y$ be involved to the 7th Power. The Powers from a Binomil have $+$ before every Term, as you see by the Involution of $x+y$.

	1	$x^7+x^6+x^5+x^4+x^3+x^2+x$
	2	$y+y^2+y^3+y^4+y^5+y^6+y^7$
$1+2$	3	$x^7+x^6y+x^5y^2+x^4y^3+x^3y^4+x^2y^5+xy^6+y^7$

now these joined, the 7th Power of $x+y$ stands in this Manner, without the Unicæ.

N. B. The leading Quantity x, decreases in Arithmetical Proportion; the other Quantity y, increases in the same Proportion.

To

To find the Unicæ, Sir ISAAC NEWTON has this Method.

$$1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}\times\frac{m-4}{5}\times\frac{m-5}{6}$$

$$\times\frac{m-6}{7},\ \&c.$$

To explain this Theorem, m is the exponent of the Power, that is m is 7 in the 7th Power, 6 in the 6th Power, &c. The Meaning is this, $1\times7-0=7\div1=7$, the first Unicæ. Then $7\times7-1=6=7\times6=42$, and $42\div2=21$ the second Unicæ. Then $21\times7-2=105\div3=35$, the third Unica. Then $35\times7-3=140\div4=35$, the fourth Unicæ. Then $35\times7-4=105\div5=21$, the fifth Unicæ. Then $21\times7-5=42\div6=7$, the 6th Unicæ, then $7\times7-6=7\div7=1$. Now the Unicæ being united to the above Powers, will stand in the following Manner.

$$x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6$$
$$+y^7$$

EVOLUTION.

AS Involution is a Multiplication, so Evolution is a Species of Division. If the Power given have no Coefficient prefixed to it, and its Index can be divided by the Index of the Root required, the Quotient will be the Index of the Root sought.

EXAMPLE.

EXAMPLE.

	1	x^6	x^6y^6	$x^6y^6r^6$
1 vv 2	2	x^3	x^3y^3	$x^3y^3r^3$
1 vv 3	3	x^2	x^2y^2	$x^2y^2r^2$
1 vv 6	4	x	xy	xyr

N. B. The Figures 1 vv 2, 1 vv 3, &c. denote the Index of the Root to be extracted.

If the given Power have Coefficients, then you must extract their Roots, as in Arithmetic.

EXAMPLE.

	1	$81x^4$	$1296x^8y^8$
1 vv 2	2	$9x^2$	$36x^4y^4$
1 vv 4	3	$3x$	$6x^2y^2$

But if the Root required cannot be truly extracted out of both the Coefficients and Indices of the given Power, then it is a Surd, and must have the Sign of the Root required prefixed to it.

EXAMPLE.

	1	x^5
1 vv 2	2	$\sqrt{x^5}$

To discover readily the Roots of all compound Powers in general, mind that if either the Sum, or difference of several Quantities be involved to any Power, there will arise so many single Powers as there are different Quantities.

EXAMPLE.

$x+y+n$ squared, it will be as follows:

$$xx+2xy+2xn+yy+2ny+nn.$$

Here is the xx, yy, nn; so that one may conclude the Root is $x+y+n$, as the Signs are affirmative. When you have extracted the Root, involve it again to see whether it amounts to the same, if not, it is a Surd, or not right evolved.

Algebraick Fractions.

ALL Operations in Quantity are performed the same as in Numbers, so that an Instance or two of every Sort will be sufficient.

REDUCTION.

TO change Fractions into one Denomination. Rule, Multiply the Denominators for a new Denominator, and every Numerator into all the Denominators but its own, for a new Numerator.

EXAMPLE.

$\frac{x}{y}+\frac{n}{r}=\frac{xr+ny}{yr}$ | $\frac{x}{y}$ $\frac{ny}{r}$ $\frac{2ae}{d}$ it is $\frac{xrd}{yrd}$ $\frac{nyyd}{yrd}$ $\frac{2aery}{yrd}$

$\frac{a}{x}$ $\frac{b}{3y}=\frac{3ay}{3xy}$ $\frac{bx}{3xy}$ | To bring Integers into Fractions. Rule, Multiply Integers into the Denominator for a new Numerator, and under place the Denominator. Example, reduce $a+c$ to a Fraction, whose Denominator is d, it will be $\frac{da+de}{d}$.

To reduce any Quantity to the Form of an improper Fraction, as $x+y$, or x, draw a Line of Seperation, and under it write 1, then it will stand thus, $\frac{x+y}{1}$, or $\frac{x}{1}$.

To reduce Fractions to its, or their loweſt Terms. Rule, expunge the Quantities that are alike in both Numerator and Denominator, and put down the Remainder.

EXAMPLE.

$$\frac{12bcd}{4cd}=\frac{3bcd}{cd}=3b.$$

ADDITION.

$\frac{a}{b}+\frac{c}{d}$ the Sum is, $\frac{ad+cb}{bd}$. Add $\frac{p}{q}+\frac{x}{s}+\frac{t}{y}+\frac{x}{u}$, the Sum is $\frac{psyu+xqyu+qstu+qsyx}{qsyu}$

SUBTRACTION.

From $\frac{p}{q}$ take $\frac{s}{r}$, it will be $\frac{pr-qs}{qr}$. $\frac{ab}{2x}-\frac{cd}{3f}$ $\frac{3abf-2cdx}{6xf}$.

MULTIPLICATION.

EXAMPLES.

$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$ $\quad x+\frac{y}{s}\times n+\frac{s}{r}=\frac{sx+y}{s}\times\frac{nr+s}{r}=\frac{nrsx+nry+ssx+sy}{sr}$ Ex. $\frac{r}{s}\times\frac{s}{t}\times\frac{t}{u}\times\frac{u}{x}\times\frac{x}{y}=\frac{rstux}{stuxy}=\frac{r}{y}$.

DIVISION.

DIVISION.

INvert your Divisor, and then × as in Multiplication, and the Product will be the Quotient desired.

EXAMPLE

Divide $\frac{r}{s}$ by $\frac{p}{q}$; place it thus: $\frac{q}{p})\frac{r}{s}(\frac{qr}{ps}$

Divide $\frac{r}{s}+\frac{t}{u}$ by $\frac{a}{b}+\frac{c}{d}$. Reduce, and Invert, and it will stand thus:

$\frac{bd}{ad+bc})\frac{ru+st}{su}(\frac{bdru+bdst}{adsu+bcsu}$ the true Quotient.

INVOLUTION.

INvolve the Numerator into itself for a new Numerator, and the Denominator into itself for a new Denominator.

EXAMPLE.

$\frac{a}{b}=\frac{aa}{bb}$, &c.

EVOLUTION.

EVOLUTION.

Evolve the Numerators and Denominators.

EXAMPLE.

$$\frac{9xxyy}{4nn} = \frac{3xy}{2n}$$

N. B. If it has no Root prefix the Sign $\sqrt{}^2$ $\sqrt{}^3$ $\sqrt{}^4$, &c.

EXAMPLE

$$\frac{xxyy}{nnn} = \frac{xy}{\sqrt{nnn}}$$

Of SURDS.

THEY are ſuch Numbers as can't be exactly expreſſed in Figures, and are called irrational Numbers.

Their Reductions were difficult, but now are almoſt uſeleſs; and decimal Arithmetic has rendered them more capable of exact Expreſſion.

Addition

Addition and Subtraction.

IF they be Homogenal, add or ſubtract the rational Part, and to their Sum or Difference join the Surd.

A D D I T I O N.

	1	$7\sqrt{xy}$	$7x\sqrt{ac}$	$x\sqrt{aa+cc}$
	2	$4\sqrt{xy}$	$3x\sqrt{ac}$	$4x\sqrt{aa+cc}$
1 + 2	3	$11\sqrt{xy}$	$10x\sqrt{ac}$	$5x\sqrt{aa+cc}$

S U B T R A C T I O N.

	1	$14\sqrt{xy}$	$12x\sqrt{cc}$	$5y\sqrt{aa+cc}$
	2	$7\sqrt{xy}$	$7x\sqrt{cc}$	$3y\sqrt{aa+cc}$
1 — 2	3	$7\sqrt{xy}$	$5x\sqrt{cc}$	$2y\sqrt{aa+cc}$

2d. If the Surds be Heterogeneal, add or ſubtract according to the Signs.

A D D I T I O N.

	1	$\sqrt{xy}$	$4x\sqrt{aa}$
	2	$\sqrt{nx}$	$3y\sqrt{zx}$
1 + 2	3	$\sqrt{xy}+\sqrt{nx}$	$4x\sqrt{aa}+3y\sqrt{xx}$

S U B T R A C T I O N.

	1	$\sqrt{xy}$
	2	$\sqrt{nx}$
1 — 2	3	$\sqrt{xy}-\sqrt{nx}$

MULTIPLICATION.

IF the Surds be Homogenal, take away the Sign and it is done; so the $\sqrt{xy}\times\sqrt{xy}=xy$ and $\sqrt{xx+aa}\times\sqrt{xx+aa}=xx+aa$.

	1	$\sqrt{x}$	$\sqrt{ba+da}$	$\sqrt{xx+yy}$
	2	$\sqrt{y}$	$\sqrt{ca}$	$\sqrt{xx-yy}$
1×2	3	$\sqrt{xy}$	$\sqrt{bcaa+cdaa}$	$\sqrt{xxxx-yyyy}$.

If rational Quantities be joined to the Surds, then multiply the rational into the rational, and the Surd into the Surd, and join the Products together.

	1	$x\sqrt{mn}$	$6xy\sqrt{mn+y}$	$15\sqrt{xy}$
	2	$3y\sqrt{x}$	$3n\sqrt{r}$	$4\sqrt{n}$
1×2	3	$3xy\sqrt{mnx}$	$18xny\sqrt{mnr+ry}$	$60\sqrt{xny}$

Division of Surd Quantities.

IF the Quantities are pure Surds of the same Kind, and can be divided off, without leaving a Remainder, divide them, and to their Quotient prefix their radical Sign.

EXAMPLES.

EXAMPLE.

	1	$\sqrt{xy}$	$\sqrt{xynn+nybb}$	$\sqrt{xxxx-yyyy}$
	2	$\sqrt{x}$	$\sqrt{yn}$	$\sqrt{xx-yy}$
$1 \div 2$	3	$\sqrt{y}$	$\sqrt{xn+bb}$	$\sqrt{xx+yy}$

If Rational Quantities are joined to Surd Quantities of the ſame Kind, divide the Rational by the Rational, if it can be done, and to the Quotient join the Quotient of the Surd, divided by the Surd, with its firſt radical Sign.

EXAMPLES.

	1	$3dx\sqrt{xyn}$	$20xyn\sqrt{ynxx+dcxx}$
	2	$3d\sqrt{y}$	$4y\sqrt{xx}$
$1 \div 2$	3	$x\sqrt{xn}$	$5xn\sqrt{yn+dc}$

Involution of Surd Quantities.

WHEN the Surds are not joined to rational Quantities, they are involved to the ſame Height as their Index denotes, by taking away their radical Sign.

EXAMPLES.

	1	$\sqrt{x}$	$\sqrt{xx+yy}$	$\sqrt{xx-yy}$
$1 \circledast 2$	2	x	$xx+yy$	$xx-yy$

When Surds are joined to rational Quantities, involve the rational Quantities to the same Height as the Index of the Surd denotes; then multiply the involved Quantities into the Surd Quantities, after the radical Sign is taken away, as before.

EXAMPLES.

	1	$x\sqrt{yy}$	$5x\sqrt{xx+yy}$
$1 \circledcirc 2$	2	$xxyy$	$25xxxx+25xxyy$
	1	$x : \sqrt[3]{xx+yy}$	
$1 \circledcirc 3$	2	$xxxxx+xxxyy$	
	1	$3x : \sqrt[3]{xx+yy}$	
$1 \circledcirc 3$	2	$27xxxxx+27xxxyy$	

EQUATIONS.

ALgebra solves Problems by Equations. In Equations we make use of five Processes.

The first brings each Side of the Equation to an entire Fraction.

The second, brings Equations into Integers, by multiplying alternately the Numerator of each Side into the Denominator of the other.

The third, brings by Transposition the several Members of the unknown Quantities to the same Side of the Equation, (*viz.*) to that Side, which after Transposition, will render them Affirmative.

The

The fourth, tranſpoſes all looſe Quantities from the Side of the unknown, to the other Side of the Equation.

The fifth, finds the unknown Quantity, by dividing the whole Equation by its Coefficient.

E X A M P L E.

	1	$\frac{2x}{3}+12=\frac{4x}{5}+6$
∴	2	$\frac{2x+36}{3}=\frac{4x+30}{5}$
2×3	3	$2x+36=\frac{12x+90}{5}$
3×5	4	$10x+180=12x+90$
$4-10x$	5	$180=2x+90$
$5-90$	6	$90=2x$
$6\div 2$	7	$45=x$

	1	$\frac{3x}{4}+5=\frac{5x}{6}+2$
∴	2	$\frac{3x+20}{4}=\frac{5x+12}{6}$
2×4	3	$3x+20=\frac{20x+48}{6}$
3×6	4	$18x+120=20x+48$
$4-18x$	5	$120=2x+48$
$5-48$	6	$72=2x$
$6\div 2$	7	$36=x$

N. B. The three Dots in the Margin ſignify Conſequently.

EXAMPLE.

	1	$\frac{7x}{8} - 5 = \frac{9x}{10} - 8$
1×8	2	$7x - 40 = \frac{72x}{10} - 64$
2×10	3	$70x - 400 = 72x - 640$
$3 + 400$	4	$70x = 72x - 240$
$4 + 240$	5	$70x + 240 = 72x$
$5 - 70x$	6	$240 = 2x$
$6 \div 2$	7	$120 = x$

	1	$\frac{5x}{9} - 8 = 74 - \frac{7x}{12}.$
$1 \quad \therefore$	2	$\frac{5x-72}{9} = \frac{888-7x}{12}$
2×9	3	$5x - 72 = \frac{7992-63x}{12}$
3×12	4	$60x - 864 = 7992 - 63x$
$4 + 63x$	5	$123x - 864 = 7992$
$5 + 864$	6	$123x = 8856$
$6 \div 123$	7	$x = 72$

	1	$\frac{x}{6} - 4 = 24 - \frac{x}{8}$
$\therefore$	2	$\frac{x-24}{6} = \frac{192-x}{8}$
1×6	3	$x - 24 = \frac{1152-6x}{8}$
3×8	4	$8x - 192 = 1152 - 6x$
$4 + 6x$	5	$14x - 192 = 1152$
$5 + 192$	6	$14x = 1344$
$6 \div 14$	7	$x = 96$

EXAMPLE.

EXAMPLES.

	1	$\frac{128}{3x-4} = \frac{216}{5x-6}$
$1 \times 3x-4$	2	$128 = \frac{648x-864}{5x-6}$
$2 \times 5x-6$	3	$640x - 768 = 648x - 864$
$3 + 864$	4	$640x + 96 = 648x$
$4 - 640x$	5	$96 = 8x$
$5 \div 8$	6	$12 = x$

	1	$\frac{xx-12}{3} = \frac{xx-4}{4}$
1×3	2	$xx - 12 = \frac{3xx-12}{4}$
2×4	3	$4xx - 48 = 3xx - 12$
$3 + 48$	4	$4xx = 3xx + 36$
$4 - 3xx$	5	$xx = 36$
5 vu 2	6	$x = \sqrt{36} = 6$

	1	$\frac{45}{2x+3} = \frac{57}{4x-5}$
$1 \times 2x+3$	2	$45 = \frac{114x+171}{4x-5}$
$2 \times 4x-5$	3	$180x - 225 = 114x + 171$
$3 + 225$	4	$180x = 114x + 396$
$4 - 114x$	5	$66x = 396$
$5 \div 66$	6	$x = 6$

EXAMPLE.

EXAMPLE.

	1	$\frac{42x}{x-2} = \frac{35x}{x-3}$
$1 \times x-2$	2	$42x = \frac{35xx - 70x}{x-3}$
$2 \times x-3$	3	$42xx - 126x = 35xx - 70x$
$3 \div x$	4	$42x - 126 = 35x - 70$
$4 + 126$	5	$42x = 35x + 56$
$5 - 35x$	6	$7x = 56$
$6 \div 7$	7	$x = 8$

	1	$\frac{5xx}{16} - 8 = 12$
1×16	2	$5xx - 128 = 192$
$2 + 128$	3	$5xx = 320$
$3 \div 5$	4	$xx = 64$
4 ɯ 2	5	$x = \sqrt{64} = 8$

Of Quadratic Equations, and their Solutions.

THere are three Forms of Quadratic Equations, as follows:

1	$xx + 8x = 240$. This is the firſt Form.
2	$xx - 8x = 48$. The ſecond Form.

N. B. 8 is the Coefficient.

In

In either of theſe Forms, this is the practical Rule Take half the known Coefficient and ſquare it; and add that Square to both Sides of the Equation, and then the Forms will ſtand thus:

1 C□	3	$xx + 8x + 16 = 256$
1 C□	4	$xx - 8x + 16 = 64$

Now Evolve thoſe Steps, to find the Value of x.

3 vv 2	5	$x + 4 = \surd\ 256 = 16$
4 vv 2	6	$x - 4 = \surd\ 64 = 8$
5 — 4	7	$x = 12$
6 + 4	8	$x = 12$

And in this Manner you muſt proceed in all Queſtions belonging to either of thoſe Forms.

|1| $16x - xx = 48$. This is the third Form.

The Rule for this Form is this. Change all the Signs in the firſt Step on both Sides the Equation, (that is, what has the Sign + before it, muſt have the Sign —, and what — muſt have more) and then the Equation will ſtand thus, putting the Square of x to the left Hand.

1 chang'd	2	$xx - 16x = -48$
2 C□	3	$xx - 16x + 64 = 16$

N. B. 64 added to —48, gives the Sum 16.

3 vv 2	4	$x - 8 = \surd 16 = 4$.
4 + 8	5	$x = 12$

Algebraical

Algebraical Problems, and Questions.

No. 1.

WHAT Numbers are thoſe whoſe Difference is 9, and their Sum is 63?

	1	$x+9=$ the greater Number.
	2	$x=$ the leaſt.
$1+2$	3	$2x+9=63$
$3-9$	4	$2x=54$
$4\div 2$	5	$x=27$ the leaſt Number.
by the 1ſt.	6	$x+9=36$ the greater Number.

Proof. $36+27=63$
$36-27=9$

No. 2.

What two Numbers are thoſe whoſe Difference is 8, and the Quotient of the greater divided by the leſs is 3?

	1	$x+8=$ the greater.
	2	$x=$ the leſs.
	3	$\frac{x+8}{x}=3$
$3\times x$	4	$x+8=3x$
$4-x$	5	$2x=8$
$5\div 2$	6	$x=4$ the leaſt Number.
by the 1ſt.	7	$x+8=12$ the greater.

Proof. $12-4=8$
$12\div 4=3$

No. 3.

No. 3.

What Number is that whose third Part exceeds its fourth Part by 9 ?

	1	x is the Number.
Then	2	$\frac{x}{3} = \frac{x}{4} + 9$
2×3	3	$x = \frac{3x}{4} + 27$
3×4	4	$4x = 3x + 108$
$4 - 3x$	5	$x = 108$ the Number.

Proof. $108 \div 3 = 36$
$108 \div 4 = 27$
And $36 - 27 = 9$

No. 4.

What Number is that whose third Part added to its fourth Part, the Number will be 35 ?

	1	x is the Number.
Then	2	$\frac{x}{3} + \frac{x}{4} = 35$
2×3	3	$x + \frac{3x}{4} = 105$
3×4	4	$4x + 3x = 420$
$\therefore$	5	$7x = 420$
$5 \div 7$	6	$x = 60$ The Number.

Proof. $60 \div 3 = 20$
$60 \div 4 = 15$
And $20 + 15 = 35$

No. 5.

What Number is that whose third Part -4, is equal to its fourth Part $+3$?

	1	x is the Number.
Then	2	$\frac{x}{3} - 4 = \frac{x}{4} + 3$
2×3	3	$x - 12 = \frac{3x}{4} + 9$
3×4	4	$4x - 48 = 3x + 36$
$4 + 48$	5	$4x = 3x + 84$
$5 - 3x$	6	$x = 84$

Proof. $84 \div 3 = 28$
$84 \div 4 = 21$
And $28 - 4 = 24$
And $21 + 3 = 24$

No. 6.

What two Numbers are those whose Difference is 5, and the Difference of their Squares is 95?

	1	$x + 5$ is the greatest Number.
	2	x is the less Number.
$1 \circledcirc 2$	3	$xx + 10x + 25$ the Square of the greater.
$2 \circledcirc 2$	4	xx the Square of the less.
$3 - 4$	5	$10x + 25 = 95$
$5 - 25$	6	$10x = 70$
$6 \div 10$	7	$x = 7$ the less
by the 1st.	8	$x + 5 = 12$ the greater.

Proof. $12 - 7 = 5$
And $12 \times 12 = 144$
And $7 \times 7 = 49$
Then $144 - 49 = 95$

No.

No. 7.

What two Numbers are those whose Sum is 60, and the greater is to the less, as 3 to 2.

	1	$3x$ is the greater.
	2	$2x$ is the less.
$1+2$	3	$5x=60$
$3\div 8$	4	$x=12$
by the 1st.	5	$3x=36$ the greater Number.
by the 2d.	6	$2x=24$ the least Number.

Proof. As $3:2::36:24$
For $2\times 36=72$
And $3\times 24=72$

No. 8.

What two Numbers are those whose Sum is 56, and the greater is to the less, as 6 to 2?

	1	$6x=$ the greater.
	2	$2x=$ the less.
$1+2$	3	$8x=56$
$3\div 8$	4	$x=7$
by the 1st.	5	$6x=42$ the greater.
by the 2d.	6	$2x=14$ the less.

No.

No. 9.

What two Numbers are those whose Difference is 84, and the greater is to the less, as 4 to 1?

	1	$4x =$ the greater Number.
	2	$x =$ the less Number.
$1 - 2$	3	$3x = 84$
$3 \div 3$	4	$x = 28$ the least.
by the 1st.	5	$4x = 112$ the greater.

Proof. As $4 : 1 :: 112 : 28$
For $1 \times 112 = 112$
And $4 \times 28 = 112$

No 10.

There is a Wall whose Length is double its Height, and its Height quintuple its Breadth, and it contains 1350 Cubic Feet, I demand the Length, Breadth, and Height of the Wall?

	1	$x =$ the Breadth.
	2	$5x =$ the Height.
	3	$10x =$ the Length.
$1 \times 2 \times 3$	4	$50xxx = 1350$
$4 \div 50$	5	$xxx = 27$
$5 \text{ vv } 3$	6	$x = 3$ the Breadth.
by the 2d.	7	$5x = 15$ the Height.
by the 3d.	8	$10x = 30$ the Length.

Proof. $3 \times 15 = 45$
And $45 \times 30 = 1350$

No. 11.

We read, that formerly between the Mountain *Thornax*, and the Town *Halix*; there was a Trench dug, whose Length was thirty Times its Breadth, and the Breadth triple its Depth, and its whole Capacity is 138240 Cubick Feet; I demand its particular Dimensions?

	1	$x =$ the Depth.
	2	$3x =$ the Breadth.
	3	$90x =$ the Length.
$1 \times 2 \times 3$	4	$270x^3 = 138240$
$4 \div 270$	5	$x^3 = 512$
4 ɯ 3	6	$x = 8$ Feet the Depth.
by the 2d,	7	$3x = 24$ Feet the Breadth.
by the 3d.	8	$90x = 720$ Feet the Length.

Proof. $8 \times 24 = 192$
And $192 \times 720 = 138240$

No. 12.

The Temple of *Æsculapius* was built in Form of a rectangled Triangle, one of whose Sides was 12 Paces, and the Sum of the other two was 36 Paces; I demand each Side seperately?

Let

Let $s = 36$, and let $n = 12$

	1	$x =$ the Hypothenuse of the Triangle.
	2	$s - x =$ the unknown Side.
	3	$n =$ the known Side.
$1 \bullet 2$	4	xx
$2 \bullet 2$	5	$ss - 2sx + xx$
$3 \bullet 2$	6	nn
Then	7	$xx = ss - 2sx + xx + nn$ by the 47 of the first of Euclid.
$7 + 2sx$	8	$xx + 2sx = ss + xx + nn$
$8 - xx$	9	$2sx = ss + nn$
$9 \div 2s$	10	$x = \frac{ss+nn}{2s} = 20$ the Hypothenuse.
by the 2d.	11	$s - x = 36 - 20 = 16$ the unknown Side.

Proof. $36 \times 36 = 1296 = ss$, and $12 \times 12 = 144 = nn$.
And $1296 + 144 = 1440 = ss + nn$; this divided by $72 = 2s$, gives 20 in the Quotient the Hypothenuse.
And $20 \times 20 = 400$, the Square of the Hypothenuse, $12 \times 12 = 144$, and $16 \times 16 = 256 + 144 = 400$

No. 13.

I am a brazen Lyon, my two Eyes, my Mouth, and the Sole of my right Foot, are so many several Pipes, they fill a Cistern, the right Eye fills it in two Days, the left in three, and the Sole of my Foot in 4, but my Mouth can fill it in 6 Hours; tell me in what Time they can fill the Cistern when they all run together?

1	$x =$	the required Time.
2	$\frac{x}{6}$	the Quantity carried into the Cistern by the Mouth.
3	$\frac{x}{48}$	the Quantity carried by the right Eye.
4	$\frac{x}{72}$	the Quantity carried by the left Eye.
5	$\frac{x}{96}$	the Quantity carried by the Sole of the right Foot.

Then by Inspection we get the following Equation.

	6	$\frac{19x}{96} + \frac{x}{72} = 1$
6×96	7	$19x + \frac{96x}{72} = 96$
7×72	8	$1368x + 96x = 6912$
$\therefore$	9	$1464x = 6912$
$9 \div 1464$	10	$x = 4, \frac{44}{61} =$ the Time required.

No.

No. 14.

What Number is that whose $\frac{x}{2}$, $\frac{x}{3}$, $\frac{x}{4}$, and $\frac{x}{6}$ exceeds itself by 1?

	1	$x =$ the Number. Then we have this
	2	Equation. $\frac{15x}{12} = x + 1$
2×12	3	$15x = 12x + 12$
$3 - 12x$	4	$3x = 12$
$4 \div 3$	5	$x = 4$. The Number sought.

No. 15.

Three Persons *A*, *B*, *C*, make a Contribution, which in the whole amounts to 76 Pounds, of which *A* contributes a certain Sum, *B* as much as *A*, and ten Pounds over, and *C* as much as *A* and *B* together; I demand their several Contributions?

	1	$x =$ *A*'s Contribution.
	2	$x + 10 =$ *B*'s Contribution.
$1 + 2$	3	$2x + 10 =$ *C*'s Contribution.
$1 + 2 + 3$	4	$4x + 20 = 76$
$4 - 20$	5	$4x = 56$
$5 \div 4$	6	$x = 14 =$ *A*'s Contribution.

Proof.	*A*'s Contribution	14
	B's $= 14 + 10 =$	24
	C's Contribution	38
		76

No.

No. 16.

One hath ſix Sons, and every one is 4 Years older than his next younger Brother, and the oldeſt is three times as old as the youngeſt. I demand their ſeveral Ages?

	1	$x =$ the Age of the youngeſt.
	2	$x + 4$ the Age of the next.
	3	$x + 8$ the Age of the next.
	4	$x + 12 =$ the Age of the next.
	5	$x + 16 =$ the Age of the next.
	6	$x + 20 =$ the Age of the oldeſt.
Then	7	$3x = x + 20$
$7 - x$	8	$2x = 20$
$8 \div 2$	9	$x = 10$

No. 17.

It is required to divide the Number 84 into two Parts, that three times one Part, may be equal to 4 times the other?

	1	$4x =$ the greater.
	2	$3x =$ the leſs.
$1 + 2$	3	$7x = 84$
$3 \div 7$	4	$x = 12$
by the 1ſt.	5	$4x = 48$ the greater.
by the 2d.	6	$3x = 36$ the leſs.

No. 18.

It is required to divide the Number 60 into two Parts, that $\frac{1}{7}$ of the one may equal $\frac{1}{8}$ of the other?

	1	$8x =$ the greater Number.
	2	$7x =$ the less.
$1 + 2$	3	$15x = 60$
$3 \div 15$	4	$x = 4$
by the 1st.	5	$8x = 32$ the greater.
by the 2d.	9	$7x = 28$ the less.

No. 19.

One has a Lease for 33 Years, and being asked how much of the Time was expired, answered, that $\frac{2}{3}$ of the Time past, was equal to $\frac{4}{5}$ of the Time to come. I demand the Time past and to come?

	1	$6x =$ the Time past.
	2	$5x =$ the Time to come.
$1 + 2$	2	$11x = 33$
$3 \div 11$	4	$x = 3$
by the 1st.	5	$6x = 18$ Time past.
by the 2d.	9	$5x = 15$ Years to come.

No. 20.

It is required to divide the Number 48 into two Parts, that one Part may be three times as much above 20, as the other wants of 20?

	1	$20 + 3x =$ the greater Part.
	2	$20 - x =$ the leſs Part.
$1 + 2$	3	$40 + 2x = 48$
$3 - 40$	4	$2x = 8$
$4 \div 2$	5	$x = 4$
by the 1ſt.	6	$20 + 3x = 32$ the greater Part.
by the 2d.	7	$20 - x = 16$ the leſs Part.

No. 21

Two Perſons, *A* and *B* engage at Play; *A* has 72 Guineas, and *B* 52 before they begin, and after a certain Number of Guineas won and loſt between them, *A* goes away with three times as many as *B*. I demand the Number won and loſt?

	1	$x =$ the Guineas won and loſt.
	2	$52 - x =$ *B*'s laſt Stock.
	3	$72 + x =$ *A*'s laſt Stock.
Then	4	$72 + x = 156 - 3x$ this is *B*'s laſt Stock $\times$ by 3
$4 + 3x$	5	$72 + 4x = 156$
$5 - 72$	6	$4x = 84$
$6 \div 4$	7	$x = 21$ the Number of Guineas won and loſt.

No. 22.

Two Perſons, *A* and *B*, ſpeaking of their Crowns, ſays *A* to *B*, give me five of your Crowns, and I ſhall then have as much as you. Says *B* to *A*, give me five of your Crowns, and then I ſhall have three times as many as you. How many had each?

	1	$x =$ *A*'s Crowns.
	2	$x + 5 =$ *A*'s when he had received five of *B*.
	3	$x + 5 =$ *B*'s when he had given *A* 5.
	4	$x + 10 =$ *B*'s Crowns at firſt.
	5	$x - 5 =$ *A*'s Crowns when he had given *B* 5.
	6	$x + 15 =$ *B*'s Crowns when he had received 5 of *A*.
Then	7	$x + 15 = 3x - 15$
$7 + 15$	8	$x + 30 = 3x$
$8 - x$	9	$2x = 30$
$9 \div 2$	10	$x = 15$ the Crowns *A* had.
by the 4th	11	$x + 10 = 25$ the Crowns *B* had.

No. 23.

One meeting a Company of Beggars, gives each four Pence, and had ſixteen Pence left; but if he had given to each ſix Pence, he would have wanted twelve Pence. I demand the Number of Beggars?

	1	$x =$ the Number of Beggars.
Then	2	$4x + 16 = 6x - 12$
$2 + 12$	3	$4x + 28 = 6x$
$3 - 4x$	4	$2x = 28$
$4 \div 2$	5	$x = 14$ the Number of Beggars.

No.

No. 24.

One had three Debtors whose particular Sums he had forgot, but he finds from his Accompts, that *A*'s and *B*'s together, amounted to 38 Pounds, *B*'s and *C*'s to 52 Pounds, and *C*'s and *A*'s to 42 Pounds. I demand the Particulars?

	1	$x = A$'s.
Then	2	$38 - x = B$'s.
And	3	$14 + x = C$'s.
$1 + 3$	4	$2x + 14 = 42 = A$' and C's.
$4 - 4$	5	$2x = 28$
$5 \div 2$	6	$x = 14 = A$'s Debt.
by the 2d.	7	$38 - x = 24 = B$'s Debt.
by the 3d.	8	$14 + x = 28 = C$'s Debt.

No. 25.

Find a Number which being multiplied by 12 and 48 added to the Product, as much may be produced as if the same Number was multiplied by 18?

	1	$x =$ the Number sought.
Then	2	$12x + 48 = 18x$
$2 - 12x$	3	$6x = 48$
$3 \div 6$	4	$x = 8$ the Number sought.

No.

No. 26.

A Man goes with a certain Sum of Money to a Tavern, where he borrows as much as he had then about him, and out of the whole ſpends eight Shillings; and thus he goes on to four Taverns ſucceſſively, borrowing as much Money as he brought thither, and out of the whole ſpending eight Shillings, till he has nothing left. What Money had he at firſt?

	1	$x =$ his Money at firſt.
	2	$2x =$ Money at firſt Tavern.
	3	$2x - 8 =$ his Money left.
	4	$4x - 16 =$ Money at ſecond Tavern.
	5	$4x - 24 =$ his Money left.
	6	$8x - 48 =$ Money at third Tavern.
	7	$8x - 56 =$ his Money left.
	8	$16x - 112 =$ Money at fourth Tavern.
Then	9	$16x - 120 = 0$
$9 + 120$	10	$16x = 120$
$10 \div 16$	11	$x = \frac{120}{16}$ 7*s.* 6*d.* his firſt Sum.

No. 27.

A Pack of Cards being laid into any Number of Heaps, ſo that the Spots on the under Card of each, added to the Number of Cards laid thereon, may make 12, by giving the Number of Heaps, and the Number of Cards left, to find all the Spots on the bottom Cards?

1	$6 = n$ the Number of Heaps.
2	$19 = r$ the Number of Cards left.
3	$52 = m$ the whole Pack.
4	$x =$ the Number of Spots required.

It is evident, that the Number of Spots on the bottom Card in each Heap, being added to the Number of Cards will make thirteen; therefore the Number of Spots and Cards together, in all the Heaps will be $13n$, from whence subtracting the Number of Spots, there will remain the Number of Cards only. So there arises this Equation.

5	$13n - x = m - r$.
6	$x = 13n - m + r = 45$ the No. of Spots.

No. 28.

A General disposes his Men in a square Form, and found he had 300 Men over; but when he thought of enlarging the Side of his Square by one Man, he found he should want 349 Men for that Purpose. I demand the Number of Men in the Army?

	1	$x =$ the Side of the Square.
	2	$xx + 300 =$ the Army.
	3	$x + 1$ the Number of Men designed for the Side of the Square.
3 ⊙ 2	4	$xx + 2x + 1$
Then	5	$xx + 2x + 1 - 349 = xx + 300$
$5 + 349$	6	$xx + 2x + 1 = 649 + xx$
$6 - xx$	7	$2x + 1 = 649$
$7 - 1$	8	$2x = 648$
$8 \div 2$	9	$x = 324$ Men in the Side of the Square.

No.

No. 29.

It is required to divide 20 Shillings into 20 Pieces, consisting only of Sixpences and half Crowns?

	1	$x =$ the half Crowns.
Then	2	$20 - x =$ the Sixpences.
	3	$5x =$ the Number of Sixpences, into which the half Crowns may be reduced.
$2 + 3$	4	$4x + 20 = 40$ the Number of Sixpences in the whole.
$4 - 20$	5	$4x = 20$
$5 \div 4$	6	$x = 5$ the Half Crowns.
by the 2d.	7	$20 - x = 15$ the Sixpences.

No. 30.

Divide 24 Shillings into 24 Pieces, consisting only of Nine-pences and Thirteen-pence Halfpenny's?

	1	$x =$ the Ninepences.
	2	$24 - x =$ the Thirteen-pence Halfpenny's.
Then	3	$18x =$ the Halfpence the Nine-pences may be reduced to.
	4	$648 - 27x$ the Halfpence the Thirteen-pence Halfpenny's may be reduced to.
$3 + 4$	5	$648 - 9x = 576$ the Halfpence in 24 Shillings.
$5 + 9x$	6	$648 = 576 + 9x$
$6 - 576$	7	$72 = 9x$
$7 \div 9$	8	$x = 8$ the Nine-pences.
by the 2d.	9	$24 - x = 16$ the Thirteen-pence Halfpenny's.

No. 31.

What two Numbers are thoſe, that the greater is three times the leſs, and the Sum of their Squares is five times the Sum of their Numbers ?

	1	$3x =$ the greater.
	2	$x =$ the leſs.
$1 + 2$	3	$4x =$ their Sum.
$1 \odot 2$	4	$9xx$
$2 \odot 2$	5	xx
$4 + 5$	6	$10xx =$ the Sum of their Squares.
Then	7	$10xx = 20x$
$7 \div 10x$	8	$x = 2$ the leſs Number.
by the 1ſt.	9	$3x = 6$ the greater Number.

No. 32.

A Gentleman hires a Servant for a Year, or twelve Months, and gives him for his Wages ſix Pounds and a Livery Coat of a certain Value; but at the End of ſeven Months the Servant obtains Leave to go away, and receives for his Wages fifty Shillings and the Livery Coat, which was his juſt Due. I demand the Value of the Coat?

	1	$x =$ the Value of the Coat.
Then	2	$x + 120 =$ the Year's Wages.
And	3	$\frac{7x+840}{12} = x + 50.$
3×12	4	$7x + 840 = 12x + 600.$
$4 - 600$	5	$7x + 240 = 12x$
$5 - 7x$	6	$5x = 240$
$6 \div 5$	7	$x = 48$ Shillings, the Value of the Coat.

No. 33.

Two Perſons, *A* and *B* travel together, *A* with 100 l. *B* with 48 l. and they meet with a Company of Robbers, who take twice as much from *A* as from *B*, and lelft *A* three times as much as they left *B*. I demand what they took from each?

	1	$x =$ the Money taken from *B*.
	2	$2x =$ the Money taken from *A*.
	3	$48 - x =$ the Money they left *B*.
	4	$100 - 2x =$ the Money they left *A*.
	5	$100 - 2x = 144 - 3x$
$5 + 3x$	6	$100 + x = 144$
$6 - 100$	7	$x = 44$ the Pounds taken from *B*.
bythe 2d.	8	$2x = 88$ the Pounds taken from *A*.

No. 34.

Divide the Number 90 into 4 ſuch Parts, that 2 being added to the firſt, ſubtracted from the ſecond, multiplied by the third, and dividing the fourth, may make them all equal.

	1	$x =$ the Number ſought.
	2	$x - 2 =$ the firſt.
	3	$x + 2 =$ the ſecond.
	4	$\frac{x}{2}$ the third.
	5	$2x =$ the fourth.
$2+3+4+5$	6	$4x + \frac{x}{2} = 90$
6×2	7	$8x + x = 180$
$7 \div 9$	8	$x = 20$ the Number ſought.

Proof.

Proof.

1	$x - 2 = 18$	which with two added $= 20$
2	$x + 2 = 22$	when two is subtracted $= 20$
3	$\frac{x}{2} = 10$	when multiplied by $2 = 20$
4	$2x = 40$	divided by $2 = 20$
	90	

No. 35.

Suppose a Fish's Head was five Foot long, and his Tail was as long as his Head and half his Body, and his Body was as long as his Head and Tail. I demand the particular Dimensions of it?

	1	$x =$ his Body.
	2	$\frac{x}{2} + 5 =$ his Tail.
	3	$5 =$ his Head.
$2 + 3$	4	$\frac{x}{2} + 10 = x$
4×2	5	$x + 20 = 2x$
$5 - x$	6	$20 = x$ the Body.
by the 2d.	7	$\frac{x}{2} + 5 = 15$ the Tail.

No. 36.

The *Spartans* hired three famous Painters, *Arcesilaus*, *Euphranio*, and *Onasia*, to adorn with Pictures the *Lesche Crotanorum*; they agreed to give *Arcesilaus* forty Shillings a Day, *Euphranio* was to have fifty, and *Onasia* sixty *per* Day, and the Work was finished in 120 Days; and when they was paid for

their Work, according to Agreement, they all three received equal Sums, I demand how many Days each Man worked according to this Proportion ?

	1	$x =$ *Onaſias*'s Time.
	2	$\frac{6x}{5} =$ *Euphranio*'s Time.
	3	$\frac{6x}{4} =$ *Arceſilaus*'s Time.
	4	$\frac{74x}{20} = 120$
4×20	5	$74x = 2400$
$5 \div 74$	6	$x = 32\frac{16}{37}$ the Time *Onaſias* worked.
by the 2d.	7	$\frac{6x}{5} = 38\frac{34}{37}$ the Time *Euphranio* work'd
by the 3d.	8	$\frac{6x}{4} = 48\frac{24}{37}$ the Time *Arceſilaus* worked.

No. 37.

A Man and his Wife found by repeated Experience, that a Barrel of Beer which laſted them both twelve Days, would laſt him in her Abſence twenty Days. I demand how long it would laſt her in his Abſence ?

	1	$x =$ the Time it will laſt her.
	2	20 Days the Time it will laſt him.
Then	3	$12 : 8 :: x : 20$
$\therefore$	4	$8x = 240$
$4 \div 8$	5	$x = 30$ the Days it laſts her in his Abſence.

No.

No. 38.

A Vintner has two Sorts of Wines, which equally mixed, will be worth fifteen Pence a Quart, but if he mix them in the Proportion of 2 to 3, that is, after the Rate of two Quarts of his beſt to three of his worſt, the Mixture will then be worth but fourteen Pence the Quart. I demand the Price of each?

	1	$x=$ the Price of a Quart of the beſt.
	2	$30-x=$ the Price of a Quart of the worſt.
1×2	3	$2x=$ the Price of 2 Quarts of the beſt.
2×3	4	$90-3x=$ the Price of 3 Quarts of the worſt
$3+4$	5	$90-x=70$ Pence, the Price of 5 Quarts when mixed.
$5+x$	6	$90=70+x$
$6-70$	7	$x=20$ Pence, the Price of a Quart of the beſt.
by the 2d.	8	$30-x=10$ Pence, the Price of a Quart of the worſt.

No. 39.

There are two Places, *A* and *B*, 154 Miles diſtant from each other, from which two Perſons ſet out at the ſame Time, with a Deſign to meet one another, one travelling at the Rate of three Miles every two Hours, and the other at the Rate of five Miles every four Hours. I want to know how long and how far each travelled before they meet?

I

	1	$x =$ the No. of Hours each travelled.
	2	$\frac{3x}{2} =$ the Miles the firſt travelled.
	3	$\frac{5x}{4} =$ the Miles the ſecond travelled.
$2 + 3$	4	$\frac{11x}{4} = 154$
4×4	5	$11x = 616$
$5 \div 11$	6	$x = 56$ the Hours each travelled.

No. 40.

One ſets out from a certain Place, and travels 7 Miles in five Hours; and eight Hours after another ſets out from the ſame Place, and travels the ſame Road at the rate of five Miles in three Hours. I demand how long and how far the firſt travelled before the ſecond overtook him?

	1	$x =$ the Hours the firſt travelled.
	2	$x - 8 =$ the Hours the ſecond travelled.
	3	$\frac{7x}{5} =$ the Miles the firſt travelled.
	4	$\frac{5x-40}{3}$ the Miles the ſecond travelled.
Then	5	$\frac{5x-40}{3} = \frac{7x}{5}$
5×3	6	$5x - 40 = \frac{21x}{5}$
6×5	7	$25x - 200 = 21x$
$7 + 200$	8	$25x = 21x + 200$
$8 - 21x$	9	$4x = 200$
$9 \div 4$	10	$x = 50$ the Hours the firſt travelled.

by the 2d.	11	$x - 8 = 42$ the Hours the second travelled
by the 3d.	12	$\frac{7x}{5} = 70$ Miles the first travelled.
by the 4th	13	$\frac{5x - 40}{3} = 70$ Miles the second travelled.

No. 41.

Certain Noblemen made a Progress for their Pleasure, every one of which carried the same Number of Pounds. The Number of Noblemen was equal to the Number of Servants each carried with him; the Number of Pounds each Nobleman carried was double the Number of Servants they all had amongst them, and the Sum of all this Money was 3456 l. I demand the Number of Noblemen, Servants, and Pounds each carried?

	1	$x =$ the Number of Noblemen.
	2	$xx =$ the Number of Servants.
	3	$2xx =$ the No. of Pounds each carried.
$3 \times x$	4	$2xxx = 3456$
$4 \div 2$	5	$xxx = 1728$
5 vv 3	6	$x = 12$ the Number of Noblemen.
by the 2d.	7	$xx = 144$ the Number of Servants.
by the 3d.	8	$2xx = 288$ the Pounds each Nobleman carried.

No. 42.

There are two Numbers which are to each other as 3 to 2, and the Sum of their Cubes is 4375. *Quere*, the two Numbers?

	1	$3x =$ the greater Number.
	2	$2x =$ the less Number.
1 ⊙ 3	3	$27xxx$
2 ⊙ 3	4	$8xxx$
$3 + 4$	5	$35xxx = 4375$
$5 \div 35$	6	$xxx = 125$
6 vv 3	7	$x = 5$
by the 1st.	8	$3x = 15$ the greater Number.
by the 2d.	9	$2x = 10$ the less Number.

No. 43.

A Clock has two Hands turning upon the same Center, one turns round in 12 Hours, the other in 16 Hours. I demand the Synodical Period of these two Hands, that is, how many Revolutions each Hand makes before the swifter Hand retakes the slower?

N. B. After the two Hands set out from the same Place, the swifter Hand must make one Revolution more than the slower, before they can come together again.

	1	$x =$ the No. of Hours in this Synodical Period.
	2	$\frac{x}{12}$ the Number of Revolutions the swifter Hand makes.
	3	$\frac{x}{16}$ the Number of Revolutions the slower Hand makes.
$2 - 3$	4	$\frac{x}{12} - \frac{x}{16} = 1$
4×12	5	$x - \frac{12x}{16} = 12$
5×16	6	$16x - 12x = 192$
∴	7	$4x = 192$

$7 \div 4$

$7 \div 4$	8	$x = 48$ the Hours in this Synodical Period.
by the 2d.	9	$\frac{x}{12} = 4$ the Number of Revolutions of the swifter Hand.
by the 3d.	10	$\frac{x}{16} = 3$ the Revolutions of the slower Hand.

No. 44.

A Person being asked what o'Clock it was, answered, what was then passed from Noon, was equal to $\frac{3}{5}$ of what was remaining till Midnight. I demand the Hour?

	1	$x =$ the Time from Noon.
	2	$12 - x =$ the Time remaining to Midnight.
	3	$\frac{36-3x}{5} = \frac{3}{5}$ of the Time remaining.
Then	4	$x = \frac{36-3x}{5}$
4×5	5	$5x = 36 - 3x$
$5 + 3x$	6	$8x = 36$
$6 \div 8$	7	$x = 4\frac{1}{2} =$ Hours from Noon.
by the 2d.	8	$12 - x = 7\frac{1}{2} =$ Hours to Midnight.

No. 45.

The Epicurean *Greeks* accounted their Thrium amongst their most delicious Dainties. It was a Kind of Cake, or Wafer, of a determinate Weight, $\frac{1}{5}$ of it was of the finest Wheat Flower; $\frac{1}{6}$ of it was of Eggs, together with an Ounce and half of Lard, and the same of Honey; to these they added a Hemina of Milk, which contained 9 Ounces; these Things mixed in this Proportion, was baked

 upon

upon a Fig-Leaf. I demand the Weight of the whole Cake, and the Weight of each Part?

	1	$x =$ the Weight of the whole Cake.
	2	$\frac{x}{3}$ the Weight of the Flower.
	3	$\frac{x}{6} =$ the Eggs.
	4	$3 =$ the Weight of Lard and Honey.
	5	$9 =$ the Weight of the Milk.
$2+3+4+5$	6	$\frac{6x}{12} + 12 = x$
6×12	7	$6x + 144 = 12x$
$7 - 6x$	8	$144 = 6x$
$8 \div 6$	9	$x = 24$ Ounces, the Weight of the whole Cake.

No 46.

A Captain being to maintain a Paſs with a certain Number of Men, of 416 Foot of Ground, placed his Men in a right Line, at equal Diſtances one from another, the Vacancies being ſufficient to receive no more than two Men a-piece; but finding by this Means he could defend but 250 Feet of Ground, he enlarged his Line, by opening his Vacancies twice as much as before, by which Means he found himſelf juſt able to make good the Paſs above-mention'd. I demand the Number of his Men?

	1	$x =$ the Number of Men.
	2	$x - 1 =$ the Number of Vacancies.
2×2	3	$2x - 2 =$ the Number of Men ſufficient to fill the Vacancies.

$1 + 3$

$1+3$	4	$3x-2=$ the Men ſufficient to fill the whole Line.
Then	5	$\frac{250}{3x-2}=$ the Quantity of Ground each Man ſtood upon.
3×2	6	$4x-4=$ the Men required to fill up the ſecond Vacancies.
$1+6$	7	$5x-4=$ the Men ſufficient to fill the whole Line.
Then	8	$\frac{416}{5x-4}$ the Quantity of Ground each Man ſtood on.
5 and 8	9	$\frac{416}{5x-4}=\frac{250}{3x-2}$
$9\times 5x-4$	10	$416=\frac{1250x-1000}{3x-2}$
$10\times 3x-2$	11	$1248x-832=1250x-1000$
$11+1000$	12	$1248x+168=1250x$
$12-1248x$	13	$2x=168$
$13\div 2$	14	$x=84$ the Number of Men required.

No. 47.

One buys a certain Number of Eggs, $\frac{1}{2}$ of which he bought at two a Penny, and the other $\frac{1}{2}$ at three a Penny; all which he afterwards ſold at 5 for two-pence, and contrary to his Expectation, loſt twelve Pence. I demand the Number he bought;

1	$x=$ the Number of Eggs bought.
2	$\frac{x}{2}=$ the Number at two a Penny.
3	$\frac{x}{2}=$ the Number at three a Penny.
4	$\frac{x}{4}=$ the Price of the former.

	5	$\frac{x}{6}$ = the Price of the latter.
	6	$\frac{2x}{5}$ = what he fold them for.
3 + 4	7	$\frac{5x}{12}$ = what he gives for the whole.
Then	8	$\frac{2x}{5} + 12 = \frac{5x}{12}$
8 × 5	9	$2x + 60 = \frac{25x}{12}$
9 × 12	10	$24x + 720 = 25x$
10 − 24x	11	$x = 720$ the Number of Eggs bought.

No. 48.

One begun the World with a certain Sum of Money, which he improved fo well by Trade, that at the End of the firft Year he had doubled his firft Stock, excepting 100 Pounds expended for the Ufe of his Family; the fame he continues every Year, doubling his laft Year's Stock, excepting 100 Pounds *per Ann.* as before, and at the End of 3 Years he found himfelf three times as rich as before he begun Trade. I demand his firft Stock?

	1	x = his firft Stock.
	2	$2x - 100$ = his Stock at the End of the firft Year.
	3	$4x - 300$ = his Stock at the End of the fecond Year.
	4	$8x - 700$ = his Stock at the End of the third Year.
Then	5	$8x - 700 = 3x$
5 + 700	6	$8x = 3x + 700$
6 − 3x	7	$5x = 700$
7 ÷ 5	8	$x = 140$ Pounds his firft Stock.

No.

No. 49.

One draws a certain Quantity of Wine out of a full Veſſel, which held 81 Gallons, and then filling it up with Water, takes a ſecond Draught of as much Wine and Water together, as before of Wine; and ſo he goes on taking an equal Quantity at every Draught, and ſtill recruiting the Veſſel with Water, whereby it happened that after the fourth Draught thus made, there remained but 16 Gallons of Wine in the Veſſel, all the reſt being Water. I demand how much he took out at a Draught?

N. B. As the whole Quantity of mixed Liquor before Drawing, is to the pure Wine before Drawing, ſo is the Quantity of mixed Liquor after Drawing to the Quantity of pure Wine after Drawing.

	1	$x =$ the Quantity of Wine in the Caſk after firſt Draught.
	2	$\frac{xx}{81}$ Wine after ſecond Draught.
	3	$\frac{xxx}{6561} =$ Wine after third Draught.
	4	$\frac{xxxx}{531441} =$ Wine after fourth Draught.
Then	5	$\frac{xxxx}{531441} = 16$
5×531441	6	$xxxx = 8503056$
6 vv 2	7	$xx = 2916$
7 vv 2	8	$x = 54 =$ the Wine in Caſk after firſt Draught.

No. 50.

Cupid complained that the Muses had taken away his Arrows, *Clio* saith he hath taken $\frac{1}{5}$, *Euterpe* $\frac{1}{12}$, *Thalia* $\frac{1}{8}$, *Erato* $\frac{1}{7}$, *Melpomene* $\frac{1}{20}$, *Terpsichore* $\frac{1}{4}$, *Polyhymnia* 30, *Urania* 105, and *Calliope*, the most spightful of them all, would have 360; so that now I have but 5 left. How many had he at first?

	1	$60x =$ the Number of his Arrows.
Then	2	$\frac{60x}{7} + \frac{60x}{8} + 500 = 25x$
2×7	3	$60x + \frac{420x}{8} + 3500 = 175x$
3×8	4	$480x + 420x + 28000 = 1400x$
	5	$1400x = 900x + 28000$
$5 - 900x$	6	$500x = 28000$
$6 \div 500$	7	$x = 56$
by the 1st.	8	$60x = 3360$ the No. of his Arrows.

No. 52.

Two Men speaking of their Pounds, says *A* to *B*, my Pounds are in Proportion to yours as 3 to 2: Yes, says *B*, and the Sum of our Pounds, is in Proportion to the Sum of the Squares of our Pounds as 1 to 13. How many Pounds had each?

	1	$3x =$ *A*'s Pounds.
	2	$2x =$ *B*'s Pounds.
1 ⊚ 2	3	$9xx =$ the Square of *A*'s.
2 ⊚ 2	4	$4xx =$ the Square of *B*'s.
$3 + 4$	5	$13xx =$ the Sum of their Squares.
$1 + 2$	6	$5x =$ the Sum of their Pounds.

I

Then

Then	7	$5x : 13xx :: 1 : 13$
$\therefore$	8	$13xx = 65x$
$8 \div 13x$	9	$x = 5$
by the 1ſt.	10	$3x = 15 =$ *A's* Pounds.
by the 2d.	11	$2x = 10 =$ *B's* Pounds.

No. 52.

A General who had fought a Battle, upon reviewing his Army, whoſe Foot was thrice the Number of his Horſe, finds that before the Battle $\frac{1}{12} - 120$ of his Foot had deſerted, and of his Horſe $\frac{1}{20} + 120$, beſides $\frac{1}{4}$ of his whole Army was ſent into Gariſons, (reckoning the Sick and Wounded) and $\frac{3}{8}$ of his Army remained, the reſt being either ſlain or taken Priſoners; now if you add 3000 to the Slain, the Sum will be equal to half the Foot he had at the Beginning. What Number of Men were in the Army?

	1	$60x =$ Foot.
	2	$20x =$ Horſe.
$1 + 2$	3	$80x =$ the whole Army.
	4	$5x - 120$ the Number of Foot that deſerted.
	5	$x + 120$ the Number of Horſe that deſerted.
	6	$20x =$ the Men in Gariſon.
	7	$30x$ the Number that remained.
$4+5+6+7$	8	$56x$
$3 - 8$	9	$24x + 3000 = 30x$

$9 - 24x$	10	$6x = 3000$
$10 \div 6$	11	$x = 500$
by the 1ſt.	12	$60x = 30000 =$ the No. of his Foot.
by the 2d.	13	$20x = 10000 =$ the No. of his Horſe.
$12 + 13$	14	$80x = 40000 =$ the whole Army.

No. 53.

One bought three Books, whoſe Prices were in Proportion as 12, 5, and 1; now if the Price of the firſt be doubled, of the ſecond trebled, and of the third quadrupled, the Sum of theſe Products will as much exceed fifty Shillings, as the Sum of the Prices of the greateſt and middle is below twenty-five Shillings. *Quere*, what did each coſt?

	1	$12x =$ the Price of the greateſt.
	2	$5x =$ the Price of the Middle.
	3	$x =$ the Price of the leaſt.
1×2	4	$24x$
2×3	5	$15x$
3×4	6	$4x$
$4+5+6$	7	$43x$
$1 + 2$	8	$17x$
Then	9	$43x - 50 = 25 - 17x$
$9 + 50$	10	$43x = 75 - 17x$
$10 + 17x$	11	$60x = 75$
$11 \div 60$	12	$x = 1\frac{1}{4}$ Shillings $= 1s.\ 3d.$ the Price of the leaſt.
by the 2d.	13	$5x = 6\frac{1}{4}$ Shillings $= 6s.\ 3d.$ the Price of the ſecond.
by the 1ſt.	14	$12x = 15$ Shillings the Price of the greateſt.

No.

No. 54.

Find a Number, which if added to itself, and the Sum multiplied by the same, and the same Number still subtracted from the Product; and lastly, the Remainder divided by the same, that it may produce 13?

	1	$x =$ the Number sought.
$1 + x$	2	$2x$
$2 \times x$	3	$2xx$
$3 - x$	4	$2xx - x$
$4 \div x$	5	$\frac{2xx - x}{x} = 13$
$5 \times x$	6	$2xx - x = 13x$
$6 + x$	7	$2xx = 14x$
$7 \div 2x$	8	$x = 7$ the Number sought.

No. 55.

Divide 36 into two such Parts that if 12 be added to the first, and 6 to the second, the former may be double the Sum of the latter?

	1	$x =$ the greater.
Then	2	$36 - x =$ the less.
$1 + 12$	3	$x + 12$
$2 + 6$	4	$42 - x$
Then	5	$\frac{x + 12}{2} = 42 - x$
5×2	6	$x + 12 = 84 - 2x$
$6 + 2x$	7	$3x + 12 = 84$
$7 - 12$	8	$3x = 72$
$8 \div 3$	9	$x = 24$ the greater.
by the 2d.	10	$36 - x = 12$ the less.

No. 56.

Divide 36 into two ſuch Parts, that if the leſs be multiplied by 12, and the greater made more by 6, the Multiplication will be twice the Addition?

	1	$x =$ the leaſt Part.
Then	2	$36 - x =$ the greater Part.
1×12	3	$12x$
$2 + 6$	4	$42 - x$
$3 \div 2$	5	$6x = 42 - x$
$5 + x$	6	$7x = 42$
$6 \div 7$	7	$x = 6$ the leaſt Part.
by the 2d.	8	$36 - x = 30$ the greater Part.

No. 57

Find the Side of a Cube whoſe Superfices is to the Solidity, as 6 to 11?

	1	$x =$ the Side of the Cube.
	2	$6xx =$ the Superfices.
	3	$xxx =$ the Solidity.
	4	$6 : 11 :: 6xx : xxx$
$\therefore$	5	$6xxx = 66xx$
$5 \div xx$	6	$6x = 66$
$6 \div 6$	7	$x = 11$ the Side of the Cube.

No. 58.

There is an Army to which if you add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of itſelf, and take away 5000, the Men remaining will be 100000. What is the Number of the Army?

	1	$12x =$ the Army.
	2	$6x = \frac{1}{2}$ the Army.
	3	$8x = \frac{2}{3}$ of the Army.
	4	$9x = \frac{3}{4}$ of the Army.
$1+2+3+4$	5	$35x$
Then	6	$35x = 105000$
$6 \div 35$	7	$x = 3000$
by the 1ſt.	8	$12x = 36000$ the No. of the Army.

No. 59.

Find the Side of a Square whoſe Area is to the Sum of the Sides, as 45 to 12?

	1	$x =$ the Side of the Square.
Then	2	$4x =$ the Sum of the Sides.
And	3	$xx =$ the Area.
Then	4	$45 : 12 :: xx : 4x$
$\therefore$	5	$12xx = 180x$
$5 \div 12x$	6	$x = 15$ the Side of the Square.

No. 60.

A Perſon being aſked how old he was, anſwered, if I quadruple $\frac{2}{3}$ of my Years, and add $\frac{1}{2}$ of them $+ 50$ to the Product, the Sum will be ſo much above 100, as the Number of my Years is now below 100. What was his Age?

	1	$6x =$ his Age.
Then	2	$16x = \frac{2}{3}$ of his Age quadrupled.
And	3	$3x + 50 = \frac{1}{2}$ his Age and 50
$2 + 3$	4	$19x + 50$
$4 - 100$	5	$19x - 50 = 100 - 6x$

$5 + 6x$	6	$25x - 50 = 100$
$6 + 50$	7	$25x = 150$
$7 \div 25$	8	$x = 6$
by the 1st.	9	$6x = 36$ his Age.

No. 61.

One being asked what Hour of the Day it was, answered, the Day at this Time is 16 Hours long, now if $\frac{1}{2}$ the Hours past, be added to $\frac{2}{3}$ of the Remainder, you will have the Hour desired, reckoning from Sun-rising. What was the Hour?

	1	$x =$ the Hours past.
Then	2	$16 - x =$ the Hours remaining.
And	3	$\frac{x}{2} + \frac{32-2x}{3} = x$
3×2	4	$x + \frac{64-4x}{3} = 2x$
4×3	5	$3x + 64 - 4x = 6x$
$5 + 4x$	6	$10x = 3x + 64$
$6 - 3x$	7	$7x = 64$
$7 \div 7$	8	$x = 9\frac{1}{7}$ the Time past.
by the 2d.	9	$16 - x = 6\frac{6}{7}$ the Time remaining.

No. 62.

Divide the Number 50 into two Parts, so that the greater Part being divided by 7, and the less multiplied by 3, the Sum of this Product, and the former Quotient may make the same Number proposed, which was 50. *Quere* the Numbers?

1	$7x =$ the greater Part.
2	$50 - 7x =$ the less Part.

Then

Then	3	$x + 150 - 21x = 50$
$3 + 21x$	4	$21x + 50 = x + 150$
$4 - 50$	5	$21x = x + 100$
$5 - x$	6	$20x = 100$
$6 \div 20$	7	$x = 5$
by the 1st,	8	$7x = 35$ the greater Part.
by the 2d.	9	$50 - 7x = 15$ the leſs Part.

No. 63.

If a Man gains 30 Crowns a Week, how much muſt he ſpend a Week to have 500 Crowns together, with the Expence of four Weeks remaining at the Year's End?

	1	$x =$ his weekly Expence.
	2	$52x =$ his yearly Expence.
Then	3	$56x + 500 = 1560$
$3 - 500$	4	$56x = 1060$
$4 \div 56$	5	$x = 18\frac{13}{14}$.

No. 64.

A Labourer, after 40 Weeks in which he had been at Work, lays up 28 Crowns, leſs the Pay of three Weeks, and finds he had expended 36 Crowns more the Pay of 11 Weeks. What Pay did he receive per Week?

	1	$x =$ his weekly Pay.
	2	$40x =$ his whole Wages.
Then	3	$28 - 3x + 36 + 11x = 40x$
$3 + 3x$	4	$28 + 36 + 11x = 43x$
$4 - 11x$	5	$64 = 32x$
$5 \div 32$	6	$x = 2$ Crowns his weekly Wages.

No. 65.

A Son aſked his Father how old he was, his Father anſwered, if you take 5 from my Years, and divide the Remainder by 8, the Quotient will be $\frac{1}{3}$ of your Age; but if you add 2 to your Age, and multiply the whole by 3, and then ſubtract 7 from the Product, you will have the Number of the Years of my Age. *Quere*, the Age of each?

	1	$8x+5=$ the Father's Age.
	2	$x=\frac{1}{3}$ of the Son's Age.
	3	$3x=$ the Son's Age.
Then	4	$9x-1=8x+5$
$4+1$	5	$9x=8x+6$
$5-8x$	6	$x=6$
by the 1ſt.	7	$8x+5=53$ Years the Age of the Father.
by the 3d.	8	$3x=18$ the Son's Age.

No. 66.

Find two Numbers, the Product whereof is 240, and triple of the greater divided by the leſs is 5?

	1	$5x=$ the greater.
	2	$3x=$ the leſs.
1×2	3	$15xx=240$
$3\div 15$	4	$xx=16$
4 uv 2	5	$x=4$
by the 1ſt.	6	$5x=20$ the greater Number.
by the 2d.	7	$3x=12$ the leſs Number.

No. 67.

Find two Numbers, to the Sum if you add 6 the whole ſhall be double the greater; and if you ſubtract 2 from their Difference, the Remainder will be half the leaſt?

	1	$2x=$ the Sum
Then	2	$x+3=$ the greater Number.
1 — 2	3	$x-3=$ the leaſt.
Then	4	$4=$ the Difference leſs $2=$half the leaſt.
And	5	$8=$ the leaſt Number.
5 and 4	6	$14=$ the greater Number.

No. 68.

Two Men have a Mind to purchaſe a Houſe valued at 1200 l. ſays *A* to *B*, if you give me $\frac{2}{3}$ of your Pounds I can pay for the Houſe alone; but, ſays *B* to *A*, if you give me $\frac{3}{4}$ of your Pounds, I can pay for it. How many Pounds had each?

	1	$4x=$*A*'s Pounds.
	2	$3x=$*B*'s Pounds.
Then	3	$6x=1200$
$3 \div 6$	4	$x=200$
by the 1ſt.	5	$4x=800$ Pounds $=$*A*'s Money.
by the 2d.	6	$3x=600$ Pounds $=$*B*'s Money.

No. 69.

Divide 100 twice into two Parts, ſo that the major Part of the firſt Diviſion may be three Times the minor Part of the ſecond Diviſion; and the major Part of the ſecond, may be double the minor Part of the firſt?

	1	$3x=$ major Part of the firſt Diviſion.
	2	$100-3x=$ minor Part of the firſt.
	3	$100-x=$ major Part of the ſecond Diviſion.
	4	$x=$ minor Part of the ſecond Diviſion.
Then	5	$100-x=200-6x$
$5+6x$	6	$100+5x=200$
$6-100$	7	$5x=100$
$7\div 5$	8	$x=20$ the minor Part of the ſecond Diviſion.
by the 3d.	9	$10-x=80$ the major Part of the ſecond Diviſion.
by the 2d.	10	$100-3x=40$ minor Part of the firſt Diviſion.
by the 1ſt.	11	$3x=60$ major Part of the firſt Diviſion.

No. 70.

Find three Numbers, ſo that the firſt and $\frac{1}{2}$ the Remainder, the ſecond and $\frac{1}{3}$ the Remainder, and the third and $\frac{1}{4}$ the Remainder, may always make 34? Let x, y, and z, be the Numbers.

	1	$x+\frac{y+z}{2}=34$
	2	$y+\frac{x+z}{3}=34$
	3	$z+\frac{x+y}{4}=34$
1×2	4	$2x+y+z=68$
2×3	5	$3y+x+z=102$
3×4	6	$4z+x+y=136$
$5-4$	7	$2y-x=34$
5×4	8	$4z+4x+12y=408$
$8-6$	9	$3x+11y=272$

$7+x$	10	$x+34=2y$
10×3	11	$3x+102=6y$
$9-11$	12	$11y-102=272-6y$
$12+6y$ &c.	13	$17y=374$
$13\div 17$	14	$y=22$ the second Number.
by the 10th.	15	$x=10$ the first Number.
by the 4th.	16	$z=26$ the third Number.

No. 71.

Three Merchants, from three several Fairs, meet together at an Inn, where they reckon up their Gains, and find them amount to 780 l. moreover, if you add the Gain of the first and second, and subtract the Gain of the third from the Sum, you will have the Gain of the first + 82 l.; but if you add the Gain of the second and third, and from the Sum subtract the Gain of the first, there remains the Gain of the third, —43 l. What was the Gain of each?

	1	$x=$ third Man's Gain.
Then	2	$698-2x=$the first Man's Gain.
	3	$780=$their whole Gain.
$1+2$	4	$698-x=$ the Sum of the first and third Man's Gain.
$3-4$	5	$82+x-$ the second Man's Gain.
$1+5$	6	$2x+82=$ the Gain of second and third.
$6-2$	7	$4x-616=x-43$
$\therefore$	8	$3x=573$
$8\div 3$	9	$x=191$ Pounds the third Man's Gain.
by the 2d.	10	$698-2x=316$ Pounds the Gain of the first Man.
by the 5th.	11	$82+x=273$ Pounds the second Man's Gain.

No. 72.

A Hare being 50 of her own Leaps before a Dog, makes 4 Leaps to the Dogs three; but two Leaps of the Dogs are as much as three Leaps of the Hare's. How many Leaps must the Dog take to catch the Hare?

	1	$4x+50=$ all the Hare leaps.
	2	$3x=$ all the Dogs leaps.
		Dogs : Hares : : Dogs.
Then	3	$2 : 3 :: 3x : \frac{9x}{2}$
$\therefore$	4	$\frac{9x}{2} = 4x + 50$
4×2	5	$9x=8x+100$
$5 - 8x$	6	$x=100$
by the 2d.	7	$3x=300=$ all the Dogs leaps.
by the 1st.	8	$4x+50=450=$ all the Hare leaps.

No. 73.

In three Bags is a certain Quantity of Pounds Sterling. The Sum in the first and second Bag is twenty Pounds; the Sum of the Pounds in the second and third Bag is forty-eight Pounds; and the Sum of the Pounds in the first and third Bag is 44 Pounds; what Number of Pounds is in each Bag? Let x, y, and z be the Bags.

	1	$x+y=20$
	2	$y+z=48$
	3	$x+z=44$
$2-3$	4	$y-x=4$
$1-4$	5	$2x=16$

$5 \div 2$	6	$x=8$
$1 - 6$	7	$y=12$
$2 - 7$	8	$z=36$

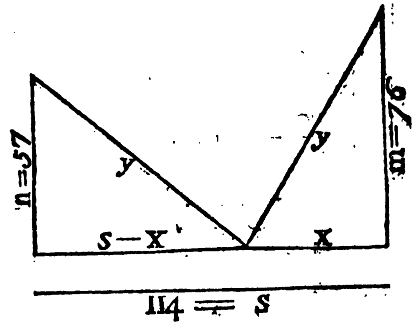
No. 74.

There are three Numbers in Geometrical Proportion, and such, if the Difference between the Sum of the Extreams and the Mean be multiplied by the Sum of the Extreams, the Product will be 1120; but if the said Differences be multiplied by the Sum of all three Proportionals, the Product will be 1456. What are the Numbers?

	1	$x=$ the Difference between the Extreams and Mean.
Then	2	$\frac{1120}{x}=$ the Sum of the Extreams.
And	3	$\frac{1120}{x}-x=$ the Mean.
$2+3$	4	$\frac{2240}{x}-x=$ the Sum of the Proportionals
$4 \times$ 1st,	5	$\frac{2240x}{x}-xx=1456$
$5 \times x$	6	$2240x-xxx=1456x$
$6+xxx$	7	$2240x=1456x+xxx$
$7 \div x$	8	$xx+1456=2240$
$8-1456$	9	$xx=784$
9 ɯ 2	10	$x=28$ the Difference betwixt the Extreams and Mean.
by the 2d.	11	$\frac{1120}{x}=40$ the Sum of Extreams.
by the 3d.	12	$\frac{1120}{x}-x=12$ the Mean.

No. 75.

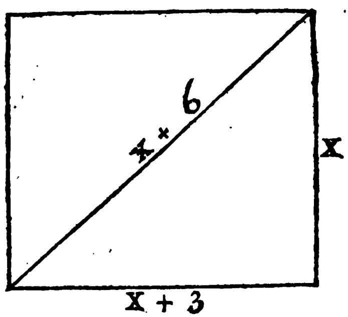
At *Thebes*, in the Street that was called *Prætides*, stood the Temple of *Euclia*, 76 Foot high, opposite to which stood the Temple of *Boedromius-Apollo*, which was 57 Foot high, and the Distance betwixt them was 114 Foot. Between these two Temples was placed the Statue of a Lyon cut in Marble, equi-distant from the Top of each Temple, which was said to be dedicated to *Hercules*, when he had defeated *Erchimus*, King of the *Orcaomenians*, I want the Distance the Lyon stood from the Base of each Temple?



	1	$x =$ the least Distance.
	2	$mm + xx = yy$ per Figure.
	3	$nn + ss - 2sx + xx = yy$ per Figure.
2 and 3	4	$mm = nn + ss - 2sx$
$4 + 2sx$	5	$mm + 2sx = nn + ss$
$5 - mm$	6	$2sx = nn + ss - mm$
$6 \div 2s$	7	$x = \frac{nn + ss - mm}{2s} = 45\frac{11}{12}$ the least Distance.
And	8	$144 - x = 68\frac{1}{12}$ the greatest.

No. 76.

The Temple of the three Graces at *Athens* ſtood upon a rectangled *Area*, whoſe Length exceeded its Breadth by three Paces, and the Diſtance from one Angle to that which was diametrically oppoſite to it, exceeded the Length alſo by three Paces. I demand its Length, Breadth, and diametrical Diſtance?



	1	$2xx + 6x + 9 = xx + 12x + 36$ per Figure.
1 $\pm$	2	$xx - 6x = 27$
2 C $\square$	3	$xx - 6x + 9 = 36$
3 ɷ 2	4	$x - 3 = 6$
4 + 3	5	$x = 9 =$ Breadth.
And	6	$x + 3 = 12 =$ Length.
And	7	$x + 6 = 15$ diametrical Diſtance.

No.

No. 77.

Buporthmos was a Promontory of *Peloponnesus*, running into the Sea, not far from the Island *Fricana*, opposite to the Gulph of *Scylla*; on this Promontory stood a Temple forty Paces long, divided into two unequal Areas; the greater was dedicated to *Ceres*, the less to her Daughter *Proserpine*; the Capacity of *Ceres*'s was 192 square Paces, *Proserpine*'s was twice as long as it was broad. What was the Breadth of this Temple?

	1	$x =$ the Breadth of the Temple.
	2	$2x =$ the Length of *Proserpine*'s.
Then	3	$40 - 2x =$ Length of *Ceres*'s,
$3 \times x$	4	$40x - 2xx = 192$
$4 \div 2$	5	$20x - xx = 96$
Then	6	$xx - 20x = -96$
$6\ C\ \square$	7	$xx - 20x + 100 = 4$
$7\ ш\ 2$	8	$x - 10 = 2$ which will be contrary to the Supposition but, it will be
	9	$x = 10 - 2 = 8$

No. 78.

In the City of *Megara*, in the Way through *Jupiter*'s Wood to the Castle *Caria*, were to be seen two little square Temples, dedicated to *Nectelius-Bacchus* and *Apostrophia-Venus*; their Pavements were laid with Stone a Foot square, but the Side of *Venus*'s Temple exceeded that of *Bacchus*'s by 12 Foot, and both the Pavements taken together, contained 2120 Stones. I demand the Length of each seperately?

No.

	1	$x =$ the Side of *Bacchus*'s Temple.
	2	$x+12=$ the Side of *Venus*'s Temple.
1 ⊙ 2	3	$xx=$ the Area of *Bacchus*'s Temple.
2 ⊙ 2	4	$xx+24x+144=$ the Area of *Venus*'s Temple.
3 + 4	5	$2xx+24x+144=2120$
5 ÷ 2	6	$xx+12x+72=1060$
6 − 72	7	$xx+12x=988$
7 C □	8	$xx+12x+36=1024$
8 ɯ 2	9	$x+6=32$
9 − 6	10	$x=26=$ the Side of *Bacchus*'s Temple.
by the 2d.	11	$x+12=38=$ the Side of *Venus*'s Temple.

No. 79.

There are 480 Men to be placed in an Oblong, whose Length and Breadth together make 52. How many is each?

	1	$x=$ the No. of Men in Length.
	2	$y=$ the No. of Men in Breadth.
Then	3	$x+y=52$
And	4	$xy=480$
3 ⊙ 2	5	$xx+2xy+yy=2704$
4 × 4	6	$4xy=1920$
5 − 6	7	$xx-2xy+yy=784$
7 ɯ 2	8	$x-y=28$
3 + 8	9	$2x=80$
9 ÷ 2	10	$x=40$
3 − 10	11	$y=12$

No. 80.

TwoCaptains diſtribute each of them 1200Crowns among a certain Number of Soldiers; one has 40 Soldiers more than the other, and its found that thoſe of the leſſer Number received five Crowns a Man more than thoſe of the greater Number. How many Soldiers had each Captain?

	1	x = the leſſer Number.
	2	$x+40$ = the greater Number.
Then	3	$\frac{1200}{x}$ = Each Man's Share in the leſs Number.
And	4	$\frac{1200}{x+40}$ = Each Man's Share in the greater Number.
$\therefore$	5	$\frac{1200}{x+40}+5=\frac{1200}{x}$
$5\times x+40$	6	$1200+5x+200=\frac{1200+48000}{x}$
$6\times x$	7	$1200x+5xx+200x=1200x+48000$
$7-1200x$	8	$5xx+200x=48000$
$8\div5$	9	$xx+40x=9600$
9 C $\square$	10	$xx+40x+400=10000$
10 ɯ 2	11	$x+20=100$
$11-20$	12	$x=80$ = the leſſer Number of Men.
by the 2d.	13	$x+40=120$ the greater Number of Men.

No. 81.

Two Parties of Soldiers have each of them an equal Number of Crowns to be diſtributed amongſt them; in one Party there were four Men more than in the other, the Money being divided, each Man in the leſs Party had eight Crowns more than thoſe of the greater, and the Number of Crowns to be diſtributed contained 172 more than the Number of Men in both Parties together. How many Men are in each Party, and what was the Number of Crowns?

	1	$x =$ the Number of Men in the leſs Party.
	2	$x + 4 =$ the Number in the greater Party.
$1 + 2$	3	$2x + 4 =$ the Number in both Parties.
$3 + 172$	4	$176 + 2x =$ the Number of Crowns.
Then	5	$\frac{176+2x}{x+4} =$ the Share of one Man in the greater Party.
And	6	$\frac{176+2x}{x} =$ the Share of one Man in the leſs Party.
$\therefore$	7	$\frac{176+2x}{x+4} + 8 = \frac{176+2x}{x}$
$7 \times x + 4$	8	$\frac{176x+2xx+8x+704}{x} = 176+2x+8x+32$
$8 \times x$	9	$176x + 2xx + 8xx + 32x = 176x + 2xx + 8x + 704$
$\therefore$	10	$8xx + 24x = 704$
$10 \div 8$	11	$xx + 3x = 88$

11 C □	12	$xx + 3x + 2,25 = 90,25$
12 vv 2	13	$x + 1,5 = \sqrt{90,25} = 9,5$
13 — 1,5	14	$x = 8$ the Number of Men in the leſs Party.
by the 2d.	15	$x + 4 = 12$ the Number of Men in the greater Party.
by the 4th.	16	$176 + 2x = 192$ the Number of Crowns each Party had.

No. 82.

There is a certain Square whoſe Side is 110 Inches, now it is required to find the Dimenſions of a rectangled Parallelogram, whoſe Perimeter ſhall be greater by four Inches, and whoſe Area ſhall be leſs by four ſquare Inches, than that of the above-mentioned Square?

	1	$12100 =$ the Area of the Square.
	2	$12096 =$ the Area of the Parallelogram.
	3	$440 =$ the Perimiter of the Square.
	4	$444 =$ the Perimeter of the Parallelogram.
Then	5	$222 = \frac{1}{2}$ the Perimeter.
	6	$x =$ the Length of the Parallelogram.
	7	$222 - x =$ the Breadth.
7×6	8	$222x - xx = 12096$ the Area.
	9	$xx - 222x = -12096$
9 C □	10	$xx - 222x + 12321 = 225$
10 vv 2	11	$x - 111 = 15$
Then	12	$x = 126$ the Length of the Parallelogram.
by the 7th.	13	$222 - x =$ the Breadth.

No.

No. 83.

Find a Number, to the Quadruple of which, if you add 91, the whole ſhall be to the Square of the Number ſought as 3 to 4?

	1	$x =$ the Number.
	2	$4x + 91$
Then	3	$4x + 91 : xx :: 3 : 4$
∴	4	$3xx = 16x + 364$
$4 - 16x$	5	$3xx - 16x = 364$
$5 \div 3$	6	$xx - \frac{16x}{3} = \frac{364}{3}$
6 C □	7	$xx - \frac{16x}{3} + \frac{64}{9} = \frac{364}{3} + \frac{64}{9} = \frac{1156}{9}$
7 w 3	8	$x - \frac{8}{3} = \sqrt{\frac{1156}{9}} = \frac{34}{3}$
$8 + \frac{8}{3}$	9	$x = \frac{42}{3} = 14$ the Number.

No. 84.

Two Merchants have a Parcel of Silk, the firſt 40 Ells, the ſecond 90; the firſt ſells for a Crown $\frac{1}{3}$ of an Ell more than the ſecond. When the Sale was over, they had taken between them 42 Crowns. How many Ells did each of them ſell for a Crown?

1	120 = the thirds of an Ell the firſt had.
2	270 = the thirds of an Ell the ſecond had.
3	$x + 1 =$ the thirds of an Ell the firſt ſells for a Crown.
4	$x =$ the thirds of an Ell the ſecond ſells for a Crown.

Then	5	$42=\frac{120}{x+1}+\frac{270}{x}$ *per* Queſtion
$5\times x+1$	6	$42x+42=120+\frac{270x+270}{x}$
$6\times x$	7	$42xx+42x=120x+270x+270$
$\therefore$	8	$42xx+42x=390x+270$
$8-390x$	9	$42xx-348x=270$
$9\div 42$	10	$xx-\frac{348x}{42}=\frac{270}{42}$
10 C □	11	$xx-\frac{348x}{42}+\frac{30276}{1764}=\frac{270}{42}+\frac{30276}{1764}$ $=\frac{41616}{1764}$
11 ɣu 2	12	$x-\frac{174}{42}=\sqrt{\frac{41616}{1764}}=\frac{204}{42}$
$12+\frac{174}{42}$	13	$x=\frac{378}{42}=9$, the thirds of an Ell the ſecond ſells.
by the 3d.	14	$x+1=10$, the thirds of an Ell the firſt ſells.

No. 85.

A Man buys a Piece of Linen, and by ſelling it again, he gains 12 Shillings, $-\frac{1}{10}$ of what he bought it for; and finds by this Means that he had gained as much for 100 Shillings as the Linen coſt him. What Price was the Linen bought and ſold at?

	1	$10x=$ the Shillings it coſt.
Then	2	$10x:12-x::100:\frac{1200-100x}{10x}$ *per* Queſtion.
$\therefore$	3	$10x=\frac{1200-100x}{10x}$

$3 \times 10x$	4	$100xx = 1200 - 100x$.
$4 + 100x$	5	$100xx + 100x = 1200$
$5 \div 100$	6	$xx + x = 12$
6 C $\square$	7	$xx + x + ,25 = 12,25$
7 vu 2	8	$x + ,5 = \sqrt{12,25} = 3,5$
$8 - ,5$	9	$x = 3$
by the 1st.	10	$10x = 30$ Shillings it cost.
$\therefore$	11	$10x + 9 = 39$ Shillings he sold it for,

No. 86.

Two Country Women *A* and *B*, carry 100 Eggs to Market, and in the Sale of them, one took as much Money as the other; but *A*, who had the best Eggs, says to *B*, had I carried as many Eggs as you, I should have had eighteen Pence for them; *B* replies, if I had brought as many Eggs as you, I should have had but eight Pence for them. How many Eggs had each?

	1	$x =$ the Number *A* carried.
	2	$100 - x =$ the Number *B* carried.
Then	3	$x : 8 :: 100 - x : \dfrac{800-8x}{x}$
And	4	$100 - x : 18 :: x : \dfrac{18x}{100-x}$
$\therefore$	5	$\dfrac{18x}{100-x} = \dfrac{800-8x}{x}$
$5 \times x$	6	$\dfrac{18xx}{100-x} = 800 - 8x$
$6 \times 100 - x$	7	$18xx = 80000 - 1600x + 8xx$
$7 - 8xx$	8	$10xx = 80000 - 1600x$
$8 \div 10$	9	$xx = 8000 - 160x$
$9 + 160x$	10	$xx + 160x = 8000$
10 C $\square$	11	$xx + 160x + 6400 = 14400$

11 vu

11 w 2	12	$x + 80 = \sqrt{14400} = 120$
12 — 80	13	$x = 40 =$ *A*'s Eggs.
by the 2d.	14	$100 - x = 60 =$ *B*'s Eggs.

No. 87.

A General is to place 969 Men, so that there may be 40 more in Depth than in Front. How many must there be in each?

	1	$x =$ the Men in Front.
	2	$x + 40 =$ the Men in Depth.
2 × 1st.	3	$xx + 40x = 969$
3 C □	4	$xx + 40x + 400 = 1369$
4 w 2	5	$x + 20 = 37$
4 — 20	6	$x = 17$ the Men in Front.
by the 2d.	7	$x + 40 = 57 =$ the Men in Depth.

No. 88.

A Merchant bought a certain Number of Yards for 70 Shillings, and found that if there had been 4 Yards more, every Yard would have stood him in two Shillings less than it did. I demand the Number of Yards?

	1	$x =$ the Number of Yards he bought
	2	$x + 4 =$ the Number had he had four Yards more.
	3	$\frac{70}{x} =$ the Price of a Yard by the first Supposition.
	4	$\frac{70}{x+4} =$ the Price of the second.

Then

Then	5	$\frac{70}{x} - 2 = \frac{70}{x+4}$
$5 \times x$	6	$70 - 2x = \frac{70x}{x+4}$
$6 \times x + 4$	7	$70x + 280 - 2xx - 8x = 70x$
$7 - 280$	8	$70x - 2xx - 8x = 70x - 280$
$8 - 70x$	9	$- 2xx - 8x = - 280$
$9 \div - 2$	10	$xx + 4x = 140$
10 C □	11	$xx + 4x + 4 = 144$
11 vv 2	12	$x + 2 = 12$
$12 - 2$	13	$x = 10$ = the Number of Yards he bought.

No. 89.

One bought a Horſe, and ſold him again for 56 Pounds, and gained as much per Cent. as the Horſe coſt. I demand the Price of the Horſe?

	1	x = the Price of the Horſe.
	2	56 = the Pounds he ſold for.
	3	$56 - x$ = the Gain.
Then	4	$x : 56 - x :: 100 : \frac{5600 - 100x}{x}$
And	5	$x = \frac{5600 - 100x}{x}$
$5 \times x$	6	$xx = 5600 - 100x$
$6 + 100x$	7	$xx + 100x = 5600$
7 C □	8	$xx + 100x + 2500 = 8100$
8 vv 2	9	$x + 50 = 90$
$9 - 50$	10	$x = 40$ = the Pounds the Horſe coſt.

No.

No. 90.

There are two Numbers whose Difference is 15, and if the Product be divided by 2, the Quotient will give the Cube of the less Number. What are the Numbers?

	1	$x =$ the less Number.
Then	2	$15 + x =$ the greater.
And	3	$xxx = \frac{xx+15x}{2}$
3×2	4	$2xxx = xx + 15x$
$4 \div x$	5	$2xx = x + 15$
$5 - x$	6	$2xx - x = 15$
$6 \div 2$	7	$xx - \frac{x}{2} = 7{,}5$
7 C □	8	$xx - \frac{x}{2} + {,}0625 = 7{,}5625$
8 ww 2	9	$x - {,}25 = 2{,}75$
$9 + {,}25$	10	$x = 3$
by the 2d.	11	$x + 15 = 18$

No. 91.

There are three Numbers in geometrical Proportion, the Difference of the Extreams is 16, and the Mean is 6. I demand the Extreams?

	1	$x =$ one of the Extreams.
	2	$x + 16 =$ the other.
Then	3	$xx + 16x = 36$
3 C □	4	$xx + 16x + 64 = 100$
4 ww 2	5	$x + 8 = 10$
$5 - 8$	6	$x = 2$
by the 2d.	7	$x + 16 = 18$

No. 92.

One buys 120 lb. of Pepper, and as many of Ginger, and received for a Crown one Pound of Ginger more than of Pepper; so that the whole Price of the Pepper came to six Crowns more than the Price of the Ginger. How many Pounds of each did he buy for a Crown?

	1	$x =$ the Pounds of Pepper for a Crown.
	2	$x + 1 =$ the Pounds of Ginger for a Crown.
	3	$\frac{120}{x}$ the Price of the Pepper.
	4	$\frac{120}{x+1}$ the Price of the Ginger.
Then	5	$\frac{120}{x+1} + 6 = \frac{120}{x}$
$5 \times x + 1$	6	$120 + 6x + 6 = \frac{120x + 120}{x}$
$6 \times x$	7	$120x + 6xx + 6x = 120x + 120$
$7 - 120x$	8	$6xx + 6x = 120$
$8 \div 6$	9	$xx + x = 20$
9 C □	10	$xx + x + ,25 = 20,25$
10 ѡ 2	11	$x + ,5 = 4,5$
$11 - ,5$	12	$x = 4$
by the 2d.	13	$x + 1 = 5$

No. 93.

A Man buys 80 lb. of Pepper, and 36 lb. of Saffron; so that for eight Crowns he had 14 Pounds of Pepper more than he had of Saffron for 26 Crowns, and what he laid out amounted to

188 Crowns. How many Pounds of Pepper had he for 8 Crowns, and how many of Saffron for 26?

	1	$x =$ Saffron.
	2	$x + 14 =$ Pepper.
Then	3	$x : 26 :: 36 : \frac{936}{x}$
And	4	$x + 14 : 8 :: 80 : \frac{640}{x+14}$
$\therefore$	5	$\frac{640}{x+14} + \frac{936}{x} = 188$
$5 \times x + 14$	6	$640 + \frac{936x + 13104}{x} = 188x + 2632$
$6 \times x$	7	$640x + 936x + 13104 = 188xx + 2632x$
$7 - 1576x$	8	$188xx + 1056x = 13104$
$8 \div 188$	9	$xx + 5,617x = 69,702$
9 C $\square$	10	$xx + 5,617x + 7,884 = 77,586$
$10 \surd\surd 2$	11	$x + 2,808 = 8,808$
$11 - 2,808$	12	$x = 6 =$ Pounds of Saffron.
by the 2d.	13	$x + 14 = 20$ Pounds of Pepper.

No. 94.

Some Companions at an Inn ſpent 3 l. 15 s. before paying the Reckoning, two went away, by which each of the reſt was obliged to pay ten Shillings more than his equal Club, I demand the Number at firſt?

	1	$x =$ the Number at firſt.
Then	2	$\frac{75}{x} =$ every Man's true Club.
And	3	$x - 2 =$ the Number of Men when two were gone.

	4	$\frac{75}{x-2}$ = Each Man's Reckoning now.
∴	5	$\frac{75}{x-2} - 10 = \frac{75}{x}$
$5 \times x-2$	6	$75 - 10x + 20 = \frac{75x - 150}{x}$
$6 \times x$	7	$75x - 10xx + 20x = 75x - 150$
$7 - 75x$	8	$-10xx + 20x = -150$
∴	9	$10xx - 20x = 150$
$9 \div 10$	10	$xx - 2x = 15$
10 C □	11	$xx - 2x + 1 = 16$
11 ѡ 2	12	$x - 1 = 4$
$12 + 1$	13	$x = 5$ the Number of Men at first.

No. 95.

Two Travellers set out at the same Time from two Cities, the one from *A*, and the other from *B*, which are 120 Miles distant from one another; the first goes five Miles a Day, and the other 3 Miles less than the Number of Days in which they meet. When will they meet?

	1	x = the Time required.
	2	$5x$ = the Miles the first travelled.
	3	$x - 3$ = the Second's daily Journey.
3×1	4	$xx - 3x$ = the Miles the second travelled.
$2 + 4$	5	$xx + 2x = 120$
5 C □	6	$xx + 2x + 1 = 121$
6 ѡ 2	7	$x + 1 = 11$
$7 - 1$	8	$x = 10$ the Days they travel.

No. 96.

A Poſt ſets out from *A* towards *B*, who travels 8 Miles a Day; after he had got 27 Miles another ſets out from *B* to meet him, who goes every Day $\frac{1}{20}$ of the whole Diſtance of the Places *A* and *B*, and meets the firſt Poſt after ſo many Days as is $\frac{1}{20}$ of the ſaid Diſtance. I demand the Number of Miles between them?

	1	$x =$ the Diſtance required.
	2	$\frac{x}{20} =$ the Time the ſecond travelled.
Then	3	$1 : 8 :: \frac{x}{20} : \frac{8x}{20}$
And	4	$\frac{8x}{20} + 27$ the Miles the firſt travelled.
Then	5	$1 : \frac{x}{20} :: \frac{x}{20} : \frac{xx}{400}$
And	6	$\frac{xx}{400} =$ the Miless the ſecond travelled.
4 + 6	7	$\frac{xx}{400} + \frac{8x}{20} + 27 = x$
7 × 400	8	$xx + \frac{3200x}{20} + 10800 = 400x$
8 × 20	9	$20xx + 3200x + 216000 = 8000x$
9 — 3200x	10	$20xx + 216000 = 4800x$
10 - 216000	11	$20xx = 4800x - 216000$
11 -- 4800x	12	$20xx - 4800x = -216000$
12 ÷ 20	13	$xx - 240x = -10800$
13 C ☐	14	$xx - 240x + 14400 = 3600$
14 vv 2	15	$x - 120 = 60$
15 + 120	16	$x = 180$ the Diſtance of the two Places.

No.

No. 97.

Two Men depart at the ſame Time, one from *London* the other from *Lincoln*, keeping the ſame Road, and when they meet, ſays *A* to *B*, I find I have travelled 20 Miles more than you, and have gone as much in $6\frac{2}{3}$ Days as you have done hitherto; I know, ſays the other, you are the beſt Walker, but if I go on as I have done, I ſhall finiſh my Journey in 15 Days hence. What was the Diſtance of the Places, and how much did they travel per Day?

	1	$x + 20 =$ the Miles gone by *A*, when they meet.
	2	$x =$ what *B* goes in the ſame Time.
$1 + 2$	3	$2x + 20 =$ the whole Diſtance.
		Thirds
Then	4	$x : 20 : x+20 : \frac{20x+400}{x}$
		Thirds
And	5	$x+20 : 45 :: x : \frac{45x}{x+20}$
∴	6	$\frac{45x}{x+20} = \frac{20x+400}{x}$
$6 \times x$	7	$\frac{45xx}{x+20} = 20x + 400$
$7 \times x+20$	8	$45xx = 20xx + 800x + 8000$
$8 - 20xx$	9	$25xx = 800x + 8000$
$9 \div 25$	10	$xx = 32x + 320$
$10 - 32x$	11	$xx - 32x = 320$
11 C □	12	$xx - 32x + 256 = 576$
12 vv 2	13	$x - 16 = \sqrt{576} = 24$
$13 + 16$	14	$x = 40$
by the 1ſt.	15	$x + 20 = 60$

14 + 15	16	$2x + 20 = 100$ the whole Distance in Miles.
And	17	*A* goes 6 Miles per Day, and *B* 4.

No. 98.

A desires *B*, to let him know how many Pounds he had in his Purse; *B* answered if I double the Number, and add 20 more than the square Root of the Sum doubled, to the doubled Sum, and lastly add this Sum to its Square, the Number of my Pounds would then be 194040. Now how many is in the Purse?

	1	$\frac{xx}{2}$ = the Pounds in the Purse.
Then	2	xx = the Pounds doubled.
And	3	$xx+x+20 = y$ by Substitution.
Then	4	$yy+y = 194040$
4 C □	5	$yy+y+,25 = 194040,25$
5 ℘ 2	6	$y+,5 = \sqrt{194040,25} = 440,5$
6 — ,5	7	$y = 440$
∴	8	$xx+x+20 = 440$ by Restitution.
8 — 20	9	$xx+x = 420$
9 C □	10	$xx+x+,25 = 420,25$
10 ℘ 2	11	$x+,5 = \sqrt{420,25} = 20,5$
11 — ,5	12	$x = 20$
by the 1st.	13	$\frac{xx}{2} = 200$ the Pounds in the Purse.

No. 99.

Two Travellers set out at the same Time from two Cities which are 129 Miles asunder, one of them goes 6 Miles every Day, and the other 2 Miles the first

first Day, $2\frac{1}{2}$ the second, 3 the third, and so on, adding $\frac{1}{2}$ Mile to every Day's Journey. In what Time will they meet with one another, and how far will each have travelled?

Then	1	$x =$ the Days each travelled.
	2	$6x =$ the Miles one travelled.
	3	$x - 1 =$ the Number of Terms.
$3 \times ,5$	4	$,5x - ,5$
	5	$,5x + 1,5 =$ the last Term.
	6	$\frac{,5xx + 3,5x}{2} + 6x = 129$
6×2	7	$,5xx + 3,5x + 12x = 258$
$\therefore$	8	$,5xx + 15,5x = 258.$
$8 \div ,5$	9	$xx + 31x = 516$
9 C □	10	$xx + 31x + 240,25 = 756,25$
10 ꝟ 2	11	$x + 15,5 = \sqrt{756,25} = 27,5$
$11 - 15,5$	12	$x = 12$ the Number of Days each travelled.

One travels 72 Miles, and the other 57.

N. B. You may see the Reason of the 3d, 4th, 5th and 6th Steps by the Theorems in Page 68.

No. 100.

Two Merchants *A* and *B* go Partners, *B* brings 420 Pounds, and *A* receives out of the Gains 52 Pounds, and the Sum of both their Shares is 854 Pounds. How much did *A* bring, and how much did *B* receive out of the Gains?

Then	1	$x =$ *A*'s Stock.
	2	$x : 52 :: 420 : \frac{21840}{x}$

And	3	$x + 420 + 52 + \frac{21840}{x} = 854$
$3 \times x$	4	$xx + 420x + 52x + 21840 = 854x$
$4 - 21840$	5	$xx + 472x = 854x - 21840$
$5 - 854x$	6	$xx - 382x = -21840$
6 C □	7	$xx - 382x + 36481 = 14641$
7 ω 2	8	$x - 191 = 121$
$8 + 191$	9	$x = 312$ *A*'s Stock.
And	10	$\frac{21840}{x} = 70$ *B*'s Gain.

No. 101.

Two Farmers sell two Sorts of Corn; *A* sells 6 Bushels, *B* receives in all 20 Crowns; now says *B* to *A*, if we add the Number of my Bushels to the Number of your Crowns, the Sum will be 28; says *A* to *B*, and if I add the Square of my Crowns to the Square of your Bushels, the Sum will be 424. How many Bushels did *B* sell, and how many Crowns did *A* receive?

	1	$x =$ *B*'s Bushels.
	2	$y =$ *A*'s Crowns.
	3	$x + y = 28$
	4	$xx + yy = 424$
3 ● 2	5	$xx + 2xy + yy = 784$
$5 - 4$	6	$2xy = 360$
$4 - 6$	7	$xx - 2xy + yy = 64$
7 ω 2	8	$x - y = 8$
$3 + 8$	9	$2x = 36$
$9 \div 2$	10	$x = 18 =$ *B*'s Bushels.
$3 - 10$	11	$y = 10 =$ *A*'s Crowns.

No.

No. 102.

A certain Man intends to travel as many Days as he has Crowns. It happens that every following Day of his Journey he had as many Crowns as he had the Day before, besides two Crowns over and above; and when he came to his Journey's End he had in all 45 Crowns. How many had he at first?

	1	$x =$ the least Term.
	2	$x =$ the Number of Terms.
Then	3	$3x - 2 =$ the greatest Term.
∴	4	$4xx - 2x = 90$
$4 \div 4$	5	$xx - ,5x = 22,5$
5 C □	6	$xx - ,5x + ,0625 = 22,5625$
6 ʍ 2	7	$x - ,25 = 4,75$
$7 + ,25$	8	$x = 5 =$ the Number of Crowns he had at first, and Days travelled.

No. 103.

A and *B* owe betwixt them 174 Pounds, *A* pays eight Pounds a Day, and *B* pays the first Day one Pound, the second Day two Pounds, the third three Pounds, and so on. In how many Days will they clear the Debt, and how much did each of them pay?

	1	$x =$ the Number of Days.
Then	2	$8x =$ what *A* pays.
And	3	$\frac{xx+x}{2} =$ what *B* pays.
$2 + 3$	4	$\frac{xx+x}{2} + 8x = 174$
4×2	5	$xx + 17x = 348$

5 C□	6	$xx + 17x + 72,25 = 420,25$
6 ɯ 2	7	$x + 8,5 = 20,5$
7 — 8,5	8	$x = 12$ the Number of Days.

No. 104.

Two Countrymen, *A* and *B*, sell their Corn at different Prices; *A* sells twenty Bushels, and *B* received for one Bushel as many Shillings as he sold Bushels. *A* perceives that if he had sold as many Bushels as *B* received Shillings, he should have received 252 Shillings, but both together received 176. How many Bushels did *B* sell, and what Price had *A*?

	1	$x =$ *B*'s Bushels.
	2	$xx =$ the Shillings *B* received.
	3	$xx : 252 :: 20 : \frac{5040}{xx}$
Then	4	$xx + \frac{5040}{xx} = 176$
4 × xx	5	$xxxx + 5040 = 176xx$
5 ±	6	$xxxx - 176xx = -5040$
6 C□	7	$xxxx - 176xx + 7744 = 2704$
7 ɯ 2	8	$xx - 88 = 52$
Then	9	$xx = 36$
9 ɯ 2	10	$x = 6$ the Bushels *B* sold.
And	11	$\frac{5040}{xx} = 140$ the Shillings *A* received.

No. 105.

Find a Number, from whose Double if you subtract 12, the Square of the Remainder -1, shall be equal to nine times the Number?

	1	$x =$ the Number.
	2	$2x - 12$
2 ⊙ 2 & $- x$	3	$4xx - 48x + 143 = 9x$
3 $\pm x$	4	$4xx - 57x = - 143$
$4 \div 4$	5	$xx - 14{,}25x = - 35{,}75$
5 C □	6	$xx - 14{,}25x + 50{,}765625 = 15{,}015625$
6 vv 2	7	$x - 7{,}125 = 3{,}875$
$7 + 7{,}125$	8	$x = 11 =$ the Number.

No. 106.

What two Numbers are thoſe whereof twice the firſt, with three times the ſecond will make ſixty; moreover twice the Square of the firſt, with three times the Square of the ſecond, will be 840?

	1	$x =$ the firſt.
	2	$y =$ the ſecond.
	3	$2x + 3y = 60$
	4	$2xx + 3yy = 840$
$3 - 2x$	5	$3y = 60 - 2x$
$5 \div 3$	6	$y = \frac{60 - 2x}{3}$
6 ⊙ 2	7	$yy = \frac{3600 - 240x + 4xx}{9}$
$4 - 2xx$	8	$3yy = 840 - 2xx$
$8 \div 3$	9	$yy = \frac{840 - 2xx}{3}$
7 & 9	10	$7560 - 18xx = 10800 - 720x + 12xx$
10 $\pm$	11	$30xx - 720x = - 3240$
$11 \div 30$	12	$xx - 24x + 144 = 36$
12 C □	13	$xx - 24x + 144 = 36$
13 vv 2	14	$x - 12 = 6$
$14 + 12$	15	$x = 18$
by the 3d.	16	$y = 8$

No.

No. 107.

A certain Number consists of two Places, which is equal to four times the Sum of its Digits, and if to the Number you add eighteen, the Digits will be inverted. What is the Number?

	1	$y =$ the Unit. Place.
	2	$x =$ the Ten's Place.
Then	3	$10x + y = 4x + 4y$
And	4	$10y + x = 4x + 4y + 18$
$3 - 4x$	5	$6x + y = 4y$
$5 - y$	6	$6x = 3y$
$6 \div 3$	7	$2x = y$
Then	8	$10y + \frac{y}{2} = 6y + 18$
8×2	9	$21y = 12y + 36$
$9 - 12y$	10	$9y = 36$
$10 \div 9$	11	$y = 4$
And	12	$21x = 12x + 18$
$11 - 12x$	13	$9x = 18$
$13 \div 9$	14	$x = 2$

No. 108.

There is a certain Rectangle whose Length exceeds its Breadth by four, and its Area is sixty. I demand the Length and Breadth?

	1	$x + 4 =$ the Length.
	2	$x =$ the Breadth.
1×2	3	$xx + 4x = 60$
3 C□	4	$xx + 4x + 4 = 64$
4 √ 2	5	$x + 2 = 8$

5—2

5 — 2	6	$x = 6$ the Breadth.
by the 1st.	7	$x + 4 = 10$ the Length.

No. 109.

There is a rectangled Parallelogram, whose Length and Breadth together equals 40, and the Area is 336. I demand the Sides?

	1	$x =$ the Length.
And	2	$40 - x =$ the Breadth.
1 × 2	3	$40x - xx = 336$
3 ±	4	$xx - 40x = -336$
4 C □	5	$xx - 40x + 400 = 64$
5 ω 2	6	$x - 20 = 8$
6 + 20	7	$x = 28$ the Length.
by the 2d.	8	$40 - x = 12$ the Breadth.

No. 110.

There is a Square whose Diameter exceeds its Side by 6, what is the Side of the Square?

	1	$x =$ the Side.
	2	$x + 6 =$ the Diameter.
Then	3	$2xx = xx + 12x + 36$
3 — xx	4	$xx = 12x + 36$
4 — $12x$	5	$xx - 12x = 36$
5 C □	6	$xx - 12x + 36 = 72$
6 ω 2	7	$x - 6 = 8{,}485$
7 + 6	8	$x = 14{,}485$ the Side of the Square.
by the 2d.	9	$x + 6 = 20{,}485$ the Diameter.

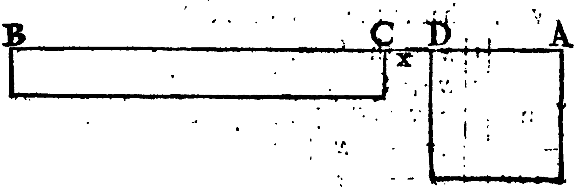
No. 111.

There is a Garden in form of a Rectangle, whose Length is three times its Breadth; this Garden is divided into two Areas, the Length of one is 18, and the Content of the Remainder is 120. I demand the Length and Breadth?

	1	$3x =$ the Length.
	2	$x =$ the Breadth.
	3	$18 =$ the Length of one Part.
	4	$3x - 18 =$ the Length of the other.
4×2	5	$3xx - 18x = 120$
$5 \div 3$	6	$xx - 6x = 40$
6 C□	7	$xx - 6x + 9 = 49$
7 ѡ 2	8	$x - 3 = 7$
$8 + 3$	9	$x = 10$ the Breadth.
by the 1st.	10	$3x = 30$ the Length.

No. 112.

Let the Line *A B* (of 70 Parts) be divided in *C*, so that $A\ C = 22$, and $B\ C$ 48, it is required to divide the same Line again in another Point, as for Example, in *D*, so that the Rectangle *B*, *D*, *C*, may = the Square *A*, *D*.

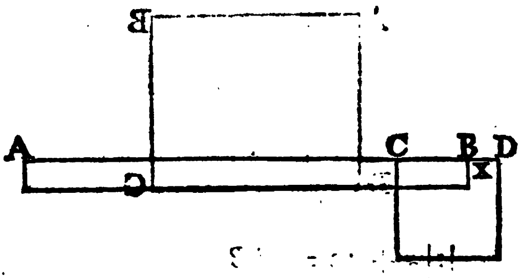


	1	$A\ C = 22$
	2	$B\ C = 48$
	3	$C\ D = x$
	4	$A\ D = 22 - x$
	5	$484 - 44x + xx = 48x$
5 ±	6	$xx - 92x = -484$
6 C□	7	$xx - 92x + 2119 = 1632$
7 ѡ 2	8	$x - 46 = 40{,}398$
	9	$x = 5{,}602$
by the 4th.	10	$22 - x = 16{,}398 = A\ D.$

No.

No. 113.

Let the Line AB be divided in C, so that $AC = 20$, and $CB = 4$, it is required to produce the Line AB to D, so that the Rectangle ABD may $=$ the Square CD?

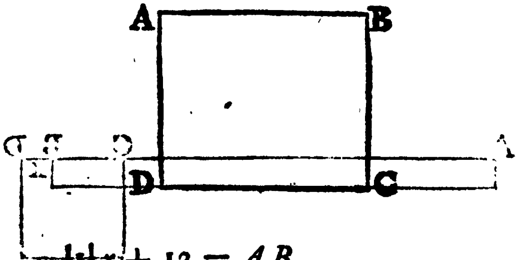


	1	$AC = 20$
	2	$CB = 4$
	3	$BD = x$
	4	$CD = 4 + x$
	5	$16 + 8x + xx = 24x$
$5 \pm$	6	$xx - 16x = -16$
$6\ C\ \Box$	7	$xx - 16x + 64 = 48$
$7\ \omega\ 2$	8	$x - 8 = 6,928$
	9	$x = 1,072$

No.

No. 114.

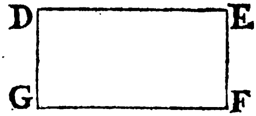
In the Rectangle *A B C D*, the Difference of the greater Side *A B*, and of the lesser Side *C D* is 12, but the Difference of the Squares of the Sides is 1680. What are the Sides?



	1	$x + 12 = AB$
	2	$x = BC$
$1 \odot 2$	3	$xx + 24x + 144$
$2 \odot 2$	4	xx
$3 - 4$	5	$24x + 144 = 1680$
$5 - 144$	6	$24x = 1536$
$6 \div 24$	7	$x = 64$
by the 1st.	8	$x + 12 = 76$

No. 115.

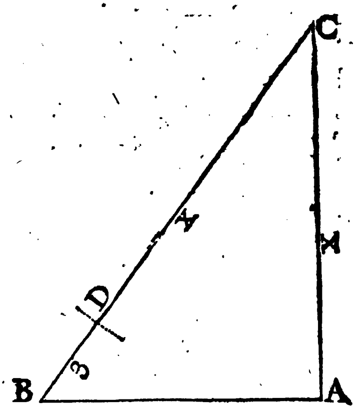
In the Rectangle *D E F G*, the Length is twice the Breadth, and the Sum of the Squares of the Length and Breadth is ten times the Sum of the Sides. *Quære* the Sides?



	1	$x = EF$
	2	$2x = DE$
Then	3	$5xx =$ the Sum of the Squares.
And	4	$30x =$ ten times the Sum of the Sides.
$\therefore$	5	$5xx = 30x$
$5 \div 5x$	6	$x = 6$
by the 2d.	7	$2x = 12$

No. 116.

In the rectangle Triangle, ABC is given the Base, $AB = 9$, and the Difference of the other Sides, that is the Segment $BD = 3$. *Quære*, the Sides AC and BC?

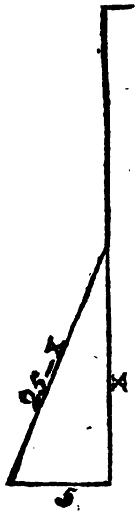


	1	$x = AC$
	2	$x + 3 = BC$
	3	$6x + 9 = 81$ per Figure.
$3 - 9$	4	$6x = 72$
$4 \div 6$	5	$x = 12 = AC$
by the 2d.	6	$x + 3 = 15 = BC$.

No. 117.

If a May-Pole be 25 Yards high, and by the Wind it is blown down, so that the Top-End struck the Ground five Yards from the Foot of the Pole, I demand the standing Part, and how much was blown down?

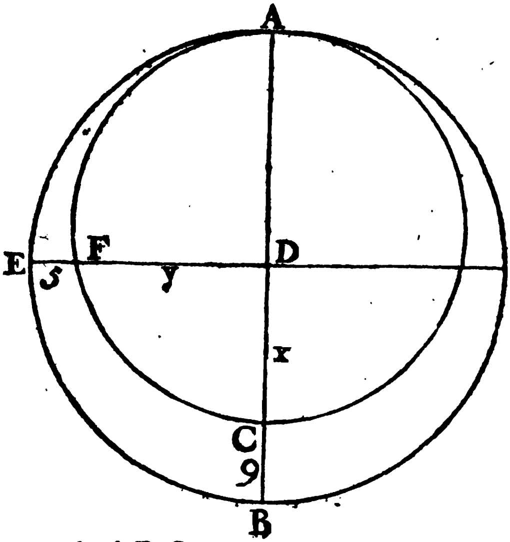
	1	$x =$ the standing Part.
	2	$25 - x =$ the Part down.
	3	$xx + 25 = 625 - 50x + xx$
$3 - xx$	4	$25 = 625 - 50x$
$4 + 50x$	5	$50x + 25 = 625$
$5 - 25$	6	$50x = 600$
$6 \div 50$	7	$x = 12$ the standing Part.
by the 2d.	8	$25 - x = 13$ Part down.



No. 118.

Let there be a Circle whose Diameter is *A B*, with another Circle whose Diameter is *A C*, touching the great Circle in the Point *A*, and from the Center of the greater Circle *D*, draw the Radius *D E*, at right Angles to *A B*, cutting the Periphery of the lesser Circle in *F*; now there is given *B C* the Difference of the Diameters $= 9$, with the Segment *E F*, equal 5. *Quære* the Diameters?

R

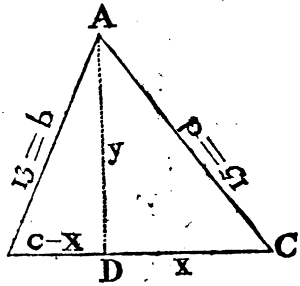


	1	$DC = x$
	2	$FD = y$
	3	$x : y :: y : \frac{yy}{x} = 9 + x$
	4	$5 + y = 9 + x$
$4 - 5$	5	$y = 4 + x$
5 ⊙ 2	6	$yy = 16 + 8x + xx$
by the 3d.	7	$9x + xx = 16 + 8x + xx$
$7 - xx$	8	$9x = 16 + 8x$
$8 - 8x$	9	$x = 16$
∴	10	$x + 9 = 25 =$ the Radius DB.
And	11	$20,5 =$ the Radius of the leaſt Circle.

No. 119.

There is an acute angled Triangle, as ABC, whoſe Sides are known, *viz.* $AB = 13 = b$, and $BC = 14 = c$, and $AC = 15 = d$. I demand the Point of the Baſe, where a Perpendicular let fall from the Vertex or Top of the Triangle, ſhall cut it and the perpendicular Line ?

	1	$x =$ Segment DC
	2	$c - x =$ Seg^t. BD.
	3	$y =$ Perpendic^r.
47 *E.* 1st	4	$dd - xx = yy$
	5	$bb - cc + 2cx - xx = yy$
4 & 5	6	$bb - cc + 2cx - xx = dd - xx$
$6 + xx$	7	$bb - cc + 2cx = dd$
$7 + cc$	8	$bb + 2cx = dd + cc$
$8 - bb$	9	$2cx = dd + cc - bb$
$9 \div 2c$	10	$x = \frac{dd + cc - bb}{2c} = 9 = DC$.
by 2d.	11	$c - x = 5$ the Segment BD.

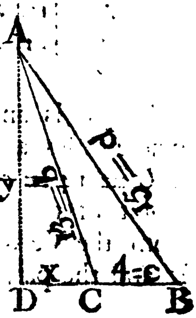


Now the Segments of the Base being found, you may find the perpendicular $AD = y$ by the fourth or fifth Steps.

No. 120.

There is an obtuse-angled Triangle, ABC, whose three Sides are known, *viz.* $AB = 15 = d$, and $AC = 13 = b$, and $BC = 4 = c$. I demand the Point without the Triangle, the Base being produced shall meet with a Perpendicular and the Perpendicular?

	1	$x = DC$ Part of the Base produced.
	2	$y = AD$ the Perpendicular.
	3	$xx + 2cx + cc + yy = dd$
47 *Eu.* 1st.	4	$xx + yy = bb$
$3 - 4$	5	$2cx + cc = dd - bb$
$5 - cc$	6	$2cx = dd - bb - cc$



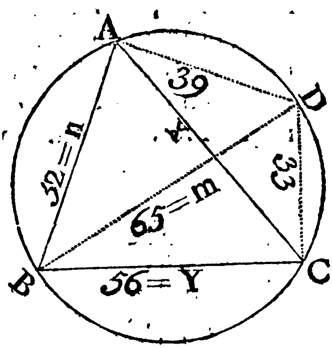
$6 \div 2c$	7	$x = \frac{dd-bb-cc}{2c} = 5 =$ the produced Part of the Base.

Now x being found, y may be found by the third or fourth Steps.

$4 - xx$	8	$yy = bb - xx$
8 ʋʋ 2	9	$y = \sqrt{bb-xx} = 12$ the Perpendicular $A\ D$.

No. 121.

There is a Triangle whereof two Sides are known, as $A\ B = 52$, and $B\ C = 56$, about the same is circumscribed a Circle, whose Diameter $= 65$. How much is the third Side of this Triangle, that is $A\ C$?



The Angles $B\ A\ D$, and $B\ C\ D$, are right Angles by 31st. *Euclid* 3d.

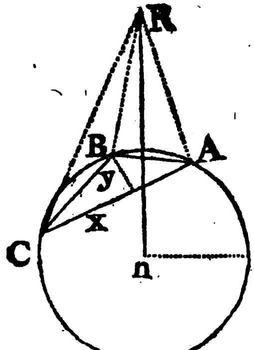
Then	1	$mm-nn=1521=$the $\square$ of $A\ D$.
And	2	$\sqrt{1521}=39=A\ D$ by 47e 1st.

	3	$mm-yy=1089=\square$ of DC.
	4	$\sqrt{1089}=33=DC$ by 47 *Euc.* 1ſt.
	5	$x=$ the Side required AC.
Then	6	$65x=3900$
$6\div 65$	7	$x=\frac{3900}{65}=60$ the Length of the Side

For in a Quadrangle inſcribed in a Circle the interior oppoſite Angles together are equal to two right Angles, by 22d *Euclid* 3d. And if there be drawn two Diagonals, the Rectangle under the Diagonals is equal to the two Rectangles under the oppoſite Sides.

No. 122.

Suppoſe a May-Pole 156 Feet high, as *N R*, and upon the Top ſits a Bird, and three Men are to ſhoot at the Bird, all of them an equal Diſtance from the Pole, and each of them diſtant from one another as follows; *B* from *C* 66 Feet, *B* from *A* 50, and *A* from *C* 104. How far muſt he ſhoot that hits the Bird?



It is evident the Places of their ſtanding make a Triangle in a Circle?

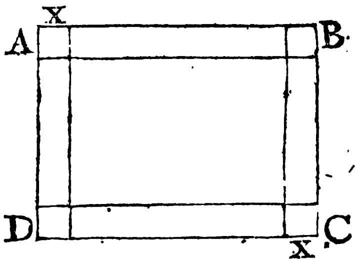
	1	$m = 66 = CB$	
	2	$n = 50 = BA$	
	3	$r = 104 = CA$	
	4	$x =$ the greater Segment.	
	5	$r - x =$ the leſs.	
	6	$y =$ the Perpendicular of the Triangle.	
Then	7	$mm - xx = yy$	} *per* 47 1ſt. *Euc.*
And	8	$nn - rr + 2rx - xx = yy$	
∴	9	$nn - rr + 2rx - xx = mm - xx$	
$9 + xx$	10	$nn - rr + 2rx = mm$	
$10 + rr$	11	$nn + 2rx = mm + rr$	
$11 - nn$	12	$2rx = mm + rr - nn$	
$12 \div 2r$	13	$x = \frac{mm + rr - nn}{2r} = 60 \frac{12}{13}$	
by the 7th.	14	$\sqrt{mm - xx} = y = 25 \frac{5}{13}$	
∴	15	$x : y :: 156 : 65$ the ſemi-Diameter of the Circle.	

Then the Square of the Pole's Height added to the Square of the ſemi-Diameter, the Sum will be 28561, whoſe Square Root is 169; and ſo many Feet is each from the Bird.

No. 123.

A Perſon hath a right-angled Piece of Land, as *A B C D*, in Length forty Yards, and in Breadth thirty Yards. This he would convert into a Garden, but finding the ſame in the Winter to be annoyed with Water, he would have the ſame raiſed one Yard over the whole Superficies. Now it is

resolved to make a Ditch round about the same which shall be two Yards deep, so that the Ditch may remain of equal Breadth and equal Depth, till so much Earth come out of the same, as will raise the whole Superficies remaining within the Ditch one Yard higher. How broad must the Ditch be?



	1	$x =$ the Breadth sought.
	2	$40 - 2x =$ the Length.
	3	$30 - 2x =$ the Breadth.
Then	4	$12xx - 420x = -1200$
$4 \div 12$	5	$xx - 35x = -100$
$5 \; C \; \square$	6	$xx - 35x + \overline{306,25} = 206,25$
$6 \; \omega\omega \; 2$	7	$x - 17,5 = \sqrt{206,25} = 14,361406$
$\therefore$	8	$x = 3,138594$ the Breadth sought.

FINIS.